St. Xavier's College – Autonomous
Mumbai

Syllabus
For EVEN Semester Courses in
MATHEMATICS
(2021 -2022)

Contents:

Theory Syllabus for Courses:

S.MAT.2.01 : Calculus – II.

S.MAT.2.02 : Algebra - II .

F.Y.B.Sc. – Mathematics **Course Code: S.MAT.2.01**

Title: CALCULUS – II

Learning Objectives: To learn about (i) Convergence of infinite series.
(ii) Limit and Continuity of real valued functions in one variable, Intermediate Value Theorem
(iii) Mean Value Theorems and other applications of real valued differentiable functions of one variable.

Number of lectures : 45

Unit I: Infinite Series (15 Lectures)

Series of real numbers, simple examples of series, Sequence of partial sums, Convergence of series, convergent and divergent series, Necessary condition: series Σa_n is convergent implies $a_n \to 0$, converse not true, Algebra of convergent series, Cauchy criterion, $\Sigma\frac{1}{n^p}$ converges for p>1, divergence of $\Sigma\frac{1}{n}$, Comparison test, limit form of comparison test, Condensation test, Alternating series, Leibnitz theorem (alternating series test) and convergence of $\Sigma\frac{(-1)^n}{n}$, Absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test and Root test (without proofs) with examples. Tests for absolute convergence.

Unit II: Limit, Continuity and Differentiation of real valued function in one variable (15 Lectures)

Limit of functions: Limit of a function, evaluation of limit of simple functions using $\in -\delta$ definition, uniqueness of limit if it exists, Algebra of limits , Limit of a composite function, Sandwich theorem, Left hand and Right hand limits, non–existence of limits, Limit as $x \to \pm\infty$.

Continuous functions and its properties: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, $\in -\delta$ definition of continuity, Sequential continuity, Algebra of continuous functions, Continuity of composite functions. Discontinuous functions, examples of removable and essential discontinuity. Intermediate value theorem and its applications, Bolzano-Weierstrass theorem (statement only), A continuous function on a closed and bounded interval is bounded and attains its bounds.

Differentiation: Definition of differentiation at a point and on an open set, examples of differentiable and non-differentiable functions, differentiable functions are continuous but not conversely, Algebra of differentiable functions, chain rule, Derivative of inverse functions, Implicit differentiation(only examples).

Unit III: Applications of differentiation (15 Lectures)

Higher order derivatives, Leibnitz rule, Rolle's theorem, Lagrange's and Cauchy's mean value theorems with applications and examples, Monotone increasing and decreasing functions with examples, L`hospital`s rule(without proof), examples of indeterminate forms, Taylor's theorem with Lagrange's form of remainder with proof, Taylor's polynomial and applications. Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphs of some standard functions, Graph of a bijective function and its inverse, Graphing of functions using first and second derivatives, concave and convex functions, points of inflection.

Reference Books

1. Robert G. Bartle and Donald R. Sherbert : Introduction to Real Analysis, Springer Verlag.
2. Ajit Kumar and Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
3. Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2000. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd.
4. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
5. Howard Anton, Calculus - A new Horizon, Sixth Edition, John Wiley and Sons Inc,1999.
6. Thomas and Finney, Calculus, 12th Edition, 2009.
7. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
8. Courant and John, An Introduction to Calculus and Analysis, Springer.

Assignments (Tutorials)

1. Calculating limit of series, Convergence tests.
2. Properties of continuous functions.
3. Differentiability, Higher order derivatives, Leibnitz theorem.
4. Mean value theorems and its applications.
5. Extreme values, increasing and decreasing functions.
6. Applications of Taylor's theorem and Taylor's polynomials.

F.Y.B.Sc. – Mathematics **Course Code: S.MAT.2.02**

Title: Algebra II

Learning Objectives: To learn about (i) System of linear equations and matrices.
(ii) Vector Spaces
(iii) Basis and linear transformations.

Number of lectures : 45

Prerequisites:

Review of vectors in R^2 and R^3 as points, Addition and scalar multiplication of vectors in terms of co-ordinates, Dot product, Scalar triple product, Length (norm) of a vector.

Unit I: System of Linear equations and Matrices **(15 Lectures)**

Parametric equation of lines and planes, System of homogeneous and non-homogeneous linear equations, the solution of system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for [m, n]=[1, 2], [1,3], [2,2], [2,3], [3,3]. Definition of n tuples of real numbers, sum of n tuples and scalar multiple of n tuple. Matrices with real entries, addition, scalar multiplication and multiplication of matrices, Transpose of a matrix, Type of matrices: zero matrix, identity matrix, scalar, diagonal, upper triangular, lower triangular, symmetric, skew-symmetric matrices, Invertible matrices, identities such as $[AB]^t = [B]^t [A]^t$, $[AB]^{-1}=[B]^{-1}[A]^{-1}$. System of linear equations in matrix form, elementary row operations, row echelon matrix, Gaussian elimination method, to deduce that the system of m homogeneous linear equations in n unknowns has a non-trivial solution if $m < n$.

Unit II: Vector spaces (15 Lectures)

Definition of real vector space, examples such as R^n with real entries, R[X]-space of m X n matrices over R, space of real valued functions on a non empty set. Subspace: Definition, examples of subspaces of R^2 and R^3 such as lines, plane passing through origin, set of 2 X 2 , 3 X 3 upper triangular, lower triangular, diagonal, symmetric and skew-symmetric matrices as subspaces of $M_2[R]$,$M_3[R]$, $P_n[X]$ of R[X] , solutions of m homogeneous linear equations in n unknowns as a subspace of R^n. Space of continuous real valued functions on a non-empty set X is a subspace of F[X,R]. Properties of subspaces such as necessary and sufficient condition for a non-empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is a subset of the other, Linear combinations of vectors in a vector space, Linear span L[S] of a non-empty subset S of a vector space, S is the generating set of L[S], linear span of a non-empty subset of a vector space is a subspace of the vector space. Linearly independent / Linearly dependent sets in a vector space, properties such as a set of vectors in a vector space is linearly dependent if and only if one of the vectors v_i is a linear combination of the other vectors v_j's.

Unit III: Basis and Linear Transformation (15 Lectures) Basis of a vector space, Dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any set of n+1 vectors in a vector space with n elements in its basis is linearly dependent, any two basis of a vector space have the same number of elements, any n linearly independent vectors in an n dimensional vector space is a basis of a vector space. If U and W are subspaces of a vector space then U+W is a subspace of the vector space, dim [U+W]=dim U + dimW – dim [U ∩ W]. Extending the basis of a subspace to a basis of a vector space. Linear transformation, kernel, matrix associated with a linear transformation, properties such as kernel of a linear transformation is a subspace of the domain space, for a linear transformation T image [T] is a subspace of the co-domain space. If V, W are vector spaces with $\{v_1, \ldots, v_n\}$ basis of V and $\{w_1, \ldots, w_n\}$ are any vectors in W then there exists a unique linear transformation T such that $T(v_i) = w_i$. Rank- nullity theorem (only statement) and examples.

Recommended Books

1. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer.
2. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India, Pvt. Ltd, 2000.

Additional Reference Books

1. M. Artin: Algebra, Prentice Hall of India Private Limited, 1991.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi, 1971.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
3. L. Smith: Linear Algebra, Springer Verlag.
4. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
5. T. Banchoff and J. Warmers: Linear Algebra through Geometry, Springer Verlag, New York, 1984.
6. Sheldon Axler: Linear Algebra done right, Springer Verlag, New York.

7. Klaus Janich: Linear Algebra.
8. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
9. Gareth Williams: Linear Algebra with Applications.

Assignments (Tutorials)

1. Solving homogeneous system of m equations in n unknowns by elimination for m , n = 1, 2, 1, 3 , 2, 2 , 2, 3 , 3, 3. Row echelon form.
2. Solving system AX= B by Gauss elimination, Solutions of system of linear equations.
3. Verifying whether V is a vector space for a given set V.
4. Linear span of a non-empty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.
5. Finding basis of a vector space such as $P_3[X]$, $M_2[R]$ etc. Verifying whether a set is a basis of a vector space. Extending basis to a basis of a finite dimensional vector space.
6. Verifying whether $T : V \rightarrow W$ is a linear transformation, finding kernel of a linear transformations and matrix associated with a linear transformation, verifying the Rank Nullity theorem.

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CIA I – 20 marks, 45 mins. (Objectives/Short questions, not more than 5 marks each)

CIA II – 20 marks, 45 mins. (Objectives/Short questions, not more than 5 marks each)

End Semester exam – 60 marks, 2 hours.

There will be three questions, one per unit. The Choice is internal- i.e. within a unit and could be between 50% to 100%