St. Xavier's College – Autonomous
Mumbai

Syllabus
For 4th Semester Courses in MATHEMATICS

(2021 - 2022)

Contents:

S.Y.B.Sc. – Mathematics Course Code: S.MAT.4.01

Title: CALCULUS IV

Learning Objectives: (i) To learn about sequences in R^n and limit, continuity, differentiability, partial/ directional derivatives, gradients of scalar fields.
(ii) To study about limits, continuity, differentiability of scalar fields.
(iii) To learn about Second derivative test for extrema of functions of two variables and the method of Lagrange`s multipliers.

Number of lectures : 45

Unit I: Functions of several variables (15 Lectures)

1. Euclidean space, R^n– norm, inner product, distance between two points, open ball in R^n, definition of an open set / neighborhood, sequences in R^n, convergence of sequences–these concepts should be specifically discussed for n = 2 and n = 3.
2. Functions from $R^n \rightarrow R$ (Scalar fields) and from $R^n \rightarrow R^n$ (Vector fields). Iterated limits, limits and continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of vector fields.
3. Directional derivatives and partial derivatives of scalar fields.
4. Mean value theorem for derivatives of scalar fields.

Reference for Unit I:
(1) T. Apostol, Calculus, Vol. 2, John Wiley.
(2) J. Stewart, Calculus, Brooke/Cole Publishing Co.

Unit II: Differentiation (15 Lectures)

1. Differentiability of a scalar field at a point (in terms of linear transformation) and in an open set, Total derivative, Uniqueness of total derivative of a differentiable function at a point. (Simple examples of finding total derivative of functions such as $f(x, y) = x^2 + y^2$, $f(x, y, z) = x + y + z$ may be taken). Differentiability at a point implies continuity, and existence of directional derivative at the point. The existence of continuous partial derivatives in neighborhood of point implies differentiability at the point.
2. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.
3. Chain rule for scalar fields.
4. Higher order partial derivatives, mixed partial derivatives. Sufficient condition for equality of mixed partial derivative.

Reference for Unit II:
(1) Calculus, Vol. 2, T. Apostol, John Wiley.
(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.

Unit III: Applications (15 Lectures)

1. Second order Taylor's formula for scalar fields.
2. Differentiability of vector fields, definition of differentiability of a vector field at a point, Hessian /Jacobian matrix, differentiability of a vector field at a point implies continuity, the chain rule for derivative of vector fields (statement only).

3. Mean value inequality.
4. Maxima, minima and saddle points.
5. Second derivative test for extrema of functions of two variables.
6. Method of Lagrange`s multipliers.

Reference for Unit III:
sections 9.9, 9.10, 9.11, 9.12, 9.13, 9.14 from T. Apostol, Calculus Vol. 2, John Wiley.

Suggested Tutorials:
1. Sequences in R^2 and R^3, limits and continuity of scalar fields and vector fields using "definition" and otherwise iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Taylor's formula, differentiation of a vector field at a point, finding Hessian/ Jacobian matrix, Mean value inequality.
6. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two/three variables and method of Lagrange`s multipliers.

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S.Y.B.Sc. – Mathematics Course Code: S.MAT.4.02

Title: ALGEBRA–IV

Learning Objectives: (i) To learn properties of groups and subgroups
(ii) To study cyclic groups and cyclic subgroups
(iii) To understand Lagrange's theorem and Group homomorphisms and isomorphisms.

Number of lectures : 45

Unit I: Groups and subgroups (15 Lectures)

(a) Definition of a group, abelian group, order of a group, finite and infinite groups.

<u>Examples of groups including</u>

(i) Z, Q, R, C under addition
(ii) Q*(=Q\{0}), R*(=R\{0}), C*(=C\{0}), Q^+ (positive rational numbers) under multiplication
(iii) Z_n – the set of residue classes modulo n under addition
(iv) U(n) – the group of prime residue classes modulo n under multiplication (v) The symmetric group S_n
(vi) The group of symmetries of plane figure.The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for n = 3, 4)
(vii) Klein 4 – group
(viii) Matrix groups $M_{\times}$(R) under addition of matrices; GL_n(R) – the set of invertible real matrices under multiplication of matrices.
(ix) Examples such as S^1 as a subgroup of C , μ – the subgroup of n^{th} roots of unity.

Properties such as

1) In a group (G, .), the following indices rules are true for all integers n, m:-
(i) $a^n a^m = a^{n+m}$ for all a in G
(ii) $(a^n)^m = a^{nm}$ for all a in G
(iii) $(ab)^n = a^n b^n$ for all a, b in G whenever ab = ba

2) In a group (G, .), the following are true:-
(i) The identity element e of G is unique.
(ii) The inverse of every element in G is unique.
(iii) $(a^{-1})^{-1} = a$
(iv) $(ab)^{-1} = b^{-1} a^{-1}$
(v) if $a^2 = e$ for every a in G then (G, .) is an abelian group
(vi) if $(aba^{-1})^n = ab^n a^{-1}$ for every a, b in G and for every integer n
(vii) if $(ab)^2 = a^2 b^2$ for every a, b in G then (G, .) is an abelian group
(viii) $(Z^*, .)$ is a group if and if n is prime

3) Properties of order of an element such as (n and m are integers)
(i) Let o(a) = n. Then $a^m = e$ if and only if n| m
(ii) If o(a) = nm then $o(a^n) = m$.
(iii) If o(a) = n then $o(a^m) = \frac{\quad}{(\ ,\ )}$ where (n, m) is GCD of n and m
(iv) $o(aba^{-1}) = o(b)$, o(ab) = o(ba)
(v) If o(a) = n, o(b) = m, ab = ba, (n, m) = 1 then o(ab) = nm.

(b) Subgroups

1) Definition, necessary and sufficient condition for a non-empty set to be a Subgroup
2) The center Z(G) of a group G is a subgroup.
3) Intersection of two (or a family of) subgroups is a subgroup.
4) Union of two subgroups is not a subgroup in general. Union of two sub groups is a subgroup if and only if one is contained in the other.
5) Let H and K are subgroups of a group G. Then HK is a subgroup of G if and only if HK = KH.

Reference for Unit I:
(1) I.N. Herstein, Topics in Algebra.
(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit II: Cyclic groups and cyclic subgroups (15 Lectures)

(a) Cyclic subgroup of a group, cyclic groups, (examples including Z, Zn, μ).
(b) Properties such as
i) Every cyclic group is abelian
ii) Finite cyclic groups, infinite cyclic groups and their generators
iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group.
iv) Subgroup of a cyclic group is cyclic.
v) In a finite group G, G = <a> if and only if o(G) = o(a).
vi) Let G = <a> and o(a) = n. Then G = < a^m > if and only if (m, n) = 1.
vii) If G is a cyclic group of order p^n and H<G, K<G then prove that either $H \subseteq K$ or $K \subseteq H$

Reference for Unit II:
(1) I.N. Herstein, Topics in Algebra.
(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit III: Lagrange's Theorem and Group homomorphism (15 Lectures) a)
Definition of a Coset and properties such as

1) If H is a subgroup of group G and x∈G then prove that
   (i) xH = Hx if and only if x∈G (ii) Hx = H if and only if x∈G
2) If H is a subgroup of group G and x, y∈G then prove that
   (i) xH = yH if and only if $x^{-1}y \in H$ (ii) Hx = Hy if and only if $xy^{-1} \in H$
3) Lagrange's theorem and consequences such as Fermat's Little theorem, Eulers's theorem.

If a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic.

b) Group homomorphisms and isomorphisms, automorphisms

1) Definition
2) Kernel and image of a group homomorphism.
3) Examples including inner automorphism.

Properties such as

(i) If f : G → G` is a group homomorphism then Ker f < G.
(ii) Let f : G → G` be a group homomorphism. Then Ker f ={e} if and only if f is 1-1.
(iii) Let f : G → G` be a group isomorphism. Then G is abelian/cyclic if and only if G` is abelian/cyclic

Reference for Unit III:

(1) I.N. Herstein, Topics in Algebra.
(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books:

1. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
2. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
5. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books:

1. S. Adhikari, An Introduction to Commutative Algebra and Number theory, Narosa Publishing House.
2. T.W. Hungerford. Algebra, Springer.
3. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luther, I.B.S. Passi. Algebra, Vol. I and II.

Suggested Tutorials:

1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle, square.
3. Subgroups.

4. Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.
6. Group homomorphisms, isomorphisms.

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S.Y.B.Sc. – Mathematics Course Code: S.MAT.4.03

Title: DIFFERENTIAL EQUATIONS

Learning Objectives:

(i) Formulate Differential Equations for various Mathematical models.
(ii) Solve higher order linear differential equations, linear system of ordinary differential equations and partial differential equations using various techniques.
(iii) Apply these techniques to solve and analyze various mathematical models.

Number of lectures: 45

Unit I: Second and Higher Order Ordinary Linear Differential equations (15 Lectures)

1. Review of First Order Ordinary Differential Equations.
2. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equation. The use of known solutions to find the general solution of homogeneous equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.
3. The homogeneous equation with constant coefficient. auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.
4. Higher Order Linear Differential Equations with constant coefficients.
5. Non-homogeneous equations: The methods of undetermined coefficients, variation of parameters, operator Method for Finding Particular Solutions.

Unit II: Linear System of Ordinary Differential Equations (15 Lectures)

1. Existence and uniqueness theorems.
2. Study of homogeneous linear system of ODEs in two variables.
3. The Wronskian W(t) of two solutions of a homogeneous linear system of ODEs in two variables,
4. Two linearly independent solutions and the general solution of a homogeneous linear system of ODEs in two variables.
5. Explicit solutions of Homogeneous linear systems with constant coefficients in two variables.
6. Solutions of non-homogeneous linear systems in two variables.

Unit III: Partial Differential Equations (15 Lectures)

1. Classification of Second Order Partial Differential Equations
2. One-Dimensional Wave Equation
    a. Vibration of an Infinite String
    b. Vibration of a Semi-infinite String
    c. Vibration of a Finite String
3. Laplace Equation
    a. Green's Function
4. Heat Conduction problem
    a. Infinite Rod Case
    b. Finite Rod Case

Recommended Books:

1. Differential equations with applications and historical notes- G. F. Simmons-McGraw Hill.
2. An introduction to ordinary differential equations - E. A. Coddington.
3. An Elementary Course in Partial Differential Equations – T. Amarnath.

Additional Reference Books:

1. Mathematical Modeling with Case Studies, A Differential Equation Approach Using Maple- Belinda Barnes and Glenn R. Fulford-Taylor and Francis.
2. Differential Equations and Boundary Value Problems: Computing and Modeling-C. H. Edwards and D. E. Penny-Pearson Education.
3. Linear Partial Differential Equation for Scientists and Engineers-Tyn Myint-U and Lokenath Debnath-Springer.
4. Partial Differential Equations: An Introduction with Mathematica and MAPLE- Ioannis P Stavroulakis and Stepan A Tersian.
5. Ordinary and Partial Differential Equations-M.D.Raisinghania-S.Chand.
6. Differential Equations-Shepley L. Ross-Wiley.

Suggested Practicals (3 practicals per week per batch):

1. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
2. Solving equations using method of undetermined coefficients and method of variation of parameters.
3. Solving Higher order Linear Differential Equations.
4. Solving a system of first order linear ODEs.
5. Determining whether a given second order linear partial differential equation is elliptic, parabolic or hyperbolic.
6. Using method of separation of variables for different equations including heat equation and Laplace equation.

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