St. Xavier's College – Autonomous
Mumbai

Syllabus
For 6th Semester Courses in MATHEMATICS

(2021 - 2022)

S.Mat.6.01 - CALCULUS VI
S.Mat.6.02 - ALGEBRA VI
S.Mat.6.03 – Topology of metric spaces II
S.Mat.6.04 – NUMERICAL METHODS II
S.Mat.6.AC – Number theory and Projects

T.Y.B.Sc. - Mathematics Course Code: S.MAT.6.01

Title of Paper: CALCULUS - VI

Learning Objectives:
After completion of the course, a student should:
(i) Learn about sequences and series of real and complex functions.
(ii) Find power series representations of functions.
(iii) Find Laurent series of a function in the neighbourhood of isolated singularity. (iv) Learn about analytic functions and understand the difference between differentiable and analytic functions.

Number of lectures: 45

Unit I: Sequences and Series of Functions (12 Lectures)

Sequence of functions-pointwise and uniform convergence of sequences of real-valued functions, examples.
Uniform convergence implies pointwise convergence, example to show converse not true. Series of functions, convergence of series of functions, Weierstrass M-test. Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval. Examples.
Consequences of these properties for series of functions, term by term differentiation and integration.

Unit II: Power Series (11 Lectures)

Limit superior and Limit inferior.
Power series in $\mathbb{R}$ centered at origin and at some point in $\mathbb{R}$, radius of convergence, region (interval) of convergence, uniform convergence, term-by-term differentiation and integration of power series, Examples.
Cauchy Hadamard Theorem. Abel's Theorem.
Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Unit III: Introduction to Complex Analysis (11 Lectures)

Review of Complex Numbers: Complex Plane, Polar Coordinates, Exponential Map, Powers and roots of Complex numbers. De Moivre's formula, $\mathbb{C}$ as a metric space, Bounded and Unbounded Sets, point at infinity-extended complex plane, Sketching of set in complex plane.
Limit at a point, theorems on limit, convergence of sequences of complex numbers and results using properties of real sequences.
Functions: $\mathbb{C} \to \mathbb{C}$, real and imaginary part of functions, continuity at a point and algebra of continuous functions.
Derivatives of: $\mathbb{C} \to \mathbb{C}$, comparison between differentiability in real and complex sense,

Cauchy-Riemann equations, sufficient condition for differentiability, analytic functions, conjugate of an analytic function is analytic, chain rule.
Harmonic functions and harmonic conjugates.
Mobius Transformations – Definition and examples.

Unit IV: Complex Power Series (11 Lectures)

Definite integrals of functions.
Cauchy Integral Formula, Derivatives of Analytic function.
Taylor's Theorem for analytic functions.
Exponential Functions and its properties, Trigonometric Functions, Hyperbolic Functions. Power series of complex numbers and related results following from Unit II, radius of convergences, disc of convergence, uniqueness of series representation, examples. Definition of Laurent series, Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, statement of residue theorem and calculation of residue.

References:

1. Richard R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi.
2. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis.
3. J. W. Brown and R. V. Churchill, Complex Variables and Applications.

Additional References:

1. Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis.
2. Charles G. Denlinger, Elements of Real Analysis, Jones and Bartlett (Student Edition),2011.
3. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable.
4. Theodore W. Gamelin, Complex analysis, Springer.
5. Joseph Bak and Donald J. Newman, Complex analysis (2nd Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.
6. Murray R. Spiegel, John Schiller and Seymour Lipschutz, Complex Variables, Schaum`s Outline series McGraw-Hill Book Company, Singapore.
7. John B. Conway, Functions of one complex variable, Springer, Second edition.
8. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, New Delhi.
9. Lars V. Ahlfors, Complex Analysis, Mc-Graw Hill Education.
10. Dennis G. Zill, Patrick Shanahan, Complex Analysis - A first course with Applications.

Suggested Practicals (3 practicals per batch per week):

1. Pointwise and uniform convergence of sequence functions, properties.
2. Point wise and uniform convergence of series of functions and properties.

3. Limit continuity and derivatives of functions of complex variables.
4. Analytic function, finding harmonic conjugate, Mobius transformations.
5. Cauchy integral formula, Taylor series, Power Series
6. Finding isolated singularities- removable, pole and essential, Laurent series, Calculation of residue.

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T.Y.B.Sc. – Mathematics Course Code: S.MAT.6.02

Title: ALGEBRA VI

Learning Objectives (i)To learn Normal Subgroups and Classification of groups.

(ii) To learn Ring theory.

Unit I. Group Theory (12L)

Review of Groups, Subgroups, Abelian groups, Order of a group, Finite and infinite groups, Cyclic groups, The Center Z(G) of a group G, Group homomorphisms, isomorphisms, automorphisms, inner automorphisms (No question be asked),Normal subgroups: Normal subgroups of a group, definition and examples including center of a group, Quotient group, Alternating group A_n, Cycles. Listing normal subgroups of A_4, S_3. First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups), Second Isomorphism theorem, third Isomorphism theorem, Cayley's theorem, External direct product of direct products, Order of an element in a direct product, criterion for direct product to be cyclic, Classification of groups of order ≤ 7.

Reference for Unit I:

(i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

Unit II. Ring Theory (11L)

Motivation: Integers & Polynomials.Definitions of a ring (The definition should include the existence of a unity element),zero divisor, unit, the multiplicative group of units of a ring. Basic Properties & examples of rings, including $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, Mn($\mathbb{R}$), $\mathbb{Q}$[X], $\mathbb{R}$[X], $\mathbb{C}$[X], $\mathbb{Z}[i]$, $\mathbb{Z}[\sqrt{2}]$, $\mathbb{Z}[\sqrt{-5}]$,$\mathbb{Z}$n. Definitions of Commutative ring, integral domain (ID), Division ring, examples. Theorem such as: A commutative ring R is an integral domain if and only if for a, b, c $\in$ R with $a \neq 0$ the relation ab = ac implies that b = c. Definitions of Subring, examples. Ring homomorphisms, Properties of ring homomorphisms, Kernel of ring homomorphism, Ideals, Operations on ideals and Quotient rings, examples. Factor theorem and First and second Isomorphism theorems for rings, Correspondence Theorem for rings. Definitions of characteristic of a ring, Characteristic of an ID. Definition of field, subfield and examples, characteristic of fields. Any field is an ID and a finite ID is a field. Definitions of characteristic of a ring, Characteristic of an ID. Definition of field, subfield and examples, characteristic of fields.

Reference for Unit II:

(i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

Unit III. Polynomial Rings and Field theory (11L)

Principal ideal, maximal ideal, prime ideal, the characterization of the prime and maximal ideals in terms of quotient rings. Polynomial rings, R[X] when R is an integral domain/ Field. Divisibility in Integral Domain, Definitions of associates, irreducible and primes. Prime (irreducible)elements in $\mathbb{R}$[X], $\mathbb{Q}$[X], $\mathbb{Z}_p$[X]. Eisenstein's criterion for irreducibility in $\mathbb{Z}$.

Reference for Unit III:

(i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.
(iii) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited

Unit IV: E.D. , P.I.D., U.F.D. (11 L)

Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique Factorization Domain (UFD). Examples of ED: Z, F[X], where F is a field, and Z[i]. (ii) An ED is a PID, a PID is a UFD. (iii) Prime (irreducible) elements in R[X], Q[X], Zp[X]. Prime and maximal ideals in R[X], Q[X]. (iv) Z[X] is a not a UFD. Prime and maximal ideals in polynomial rings. Characterization of fields in terms of maximal ideals, irreducible polynomials. Construction of quotient field of an integral domain (Emphasis on Z, Q). A field contains a subfield isomorphic to Z_p or Q.

Reference for Unit IV:

(i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi. (
iii) P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Abstract
Algebra, Second edition, Foundation Books, New Delhi, 1995.

Recommended Books:

i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.
(iii) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited
(iv) P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Abstract
Algebra, Second edition, Foundation Books, New Delhi, 1995

Additional Reference Books

(i) M. Artin, Algebra, Prentice Hall of India, New Delhi.
(ii) J. B. Fraleigh, A First course in Abstract Algebra, Third edition,
Narosa, New Delhi
(iii) D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
(iv)T.W. Hungerford. Algebra, Springer.
(v) I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.

Suggested Practicals:

1) Rings, Subrings, Ideals, Ring Homomorphism and Isomorphism

2) Prime Ideals and Maximal Ideals

3) Polynomial Rings , Fields

4) E.D, P.I.D,U.F.D.

5) Normal Subgroups and quotient groups.

6) Cayley's Theorem and external direct product of groups.

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Topology of metric spaces II

Course Code : S.MAT.6.03

Course Objective : Introduction to completeness and connectedness in metric spaces.

Unit I. Complete Metric Spaces (10 L)

Definition of complete metric spaces, Examples of complete metric spaces.
Completeness property in subspaces. Nested Interval theorem in $\mathbb{R}$. Cantor's intersection theorem.

Unit II. Applications Of Cantor's Intersection theorem (15 L)

Cantor's intersection theorem. Applications of Cantor's intersection theorem:

The set of real numbers is uncountable. Density of rational numbers in real numbers.

Bolzano- Weierstrass Theorem: Every bounded sequence of real numbers has a convergent subsequence.
Intermediate Value Theorem : Let f :[a, b] $\rightarrow \mathbb{R}$ be continuous and assume that f (a) and f(b) have opposite signs, say f(a)< 0 and f(b)>0. Then there exists c (a, b) such that f(c) =0.

Let I=[a,b] be a closed and bounded interval. Let $\{J_{\alpha} : \alpha \in \Lambda\}$ be a family of open intervals such that $I \subset \bigcup_{\alpha \in \Lambda} J_\alpha$. Then there exists a finite subset $F \subset \Lambda$ such that $I \subset \bigcup_{\alpha \in F} J_\alpha$, that is, I is contained in the union of a finite number of open intervals of the given family. Finite intersection property of closed sets for compact metric space, hence every compact metric space is complete.

Unit III. Continuous functions on metric spaces (10 L)

Epsilon-delta definition of continuity at a point of a function from one metric space to another, Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples. Algebra of continuous real valued functions on a metric space, Continuity of composite of continuous functions, Continuous image of compact set is compact. Uniform continuity in a metric space, definition and examples (emphasis on $\mathbb{R}$). Contraction mapping and fixed point theorem, Applications.

Unit IV. Connected sets (10 L)

Separated sets- definition and examples, disconnected sets, disconnected and connected metric spaces, connected subsets of a metric space. Connected subsets of $\mathbb{R}$, A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected, Characterization of a connected space, viz. A metric space is connected if and only if every continuous function from X to {-1, 1} is a constant function. Path connectedness, definition and examples, A path connected subset is connected, convex sets are path connected, Connected components, An example of a connected set which is not path connected.

Reference Books:

1. S. Kumaresan, Topology of Metric spaces.
2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.

Additional Reference Books.

1. W. Rudin, Principles of Mathematical Analysis.
2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974
3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.
4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.
5. P. K. Jain. K. Ahmed. Metric Spaces. Narosa, New Delhi, 1996.
6. W. Rudin. Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.
7. D. Somasundaram, B. Choudhary. A first Course in Mathematical Analysis. Narosa, New Delhi
8. G.F. Simmous, Introduction to Topology and Modern Analysis, McGraw-Hii, New York, 1963.
9. Sutherland. Topology

Reference for Units I and II: Expository articles of MTTS programme

Suggested Practicals:

1) Examples of complete metric spaces
2) Cantor's theorem and applications
3) Continuous functions on metric spaces

4) Uniform continuity and fixed point theorem
5) Examples of connected sets and connected metric spaces
6) Path connectedness, convex sets , equivalent condition for connected set using continuous function

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T.Y. B.Sc. Mathematics Course Code: S.Mat.6.04

Title: Numerical Methods-II

Learning Objectives: To learn about (i) different interpolation methods.
(ii) polynomial approximations.
(iii) numerical differentiation and Numerical Integration
(iv) solving Ordinary Differential Equations(IVP)

Number of lectures: 45

Unit I. Interpolation (12 L)

Interpolating polynomials, Uniqueness of interpolating polynomials. Linear, Quadratic and higher order interpolation. Lagrange's Interpolation. Newton`s divided interpolation. Finite difference operators: Shift operator, forward, backward and central difference operator, Average operator and relation between them. Difference table, Relation between difference and derivatives. Fundamental theorem of difference calculus. Factorial notation. Interpolating polynomials using finite differences Gregory-Newton forward difference interpolation, Gregory-Newton backward difference interpolation, Stirling's Interpolation. Results on interpolation error.

Unit II. Polynomial Approximations and Numerical Differentiation (12 L)

Hermite Interpolation, Piecewise Interpolation: Linear, Quadratic and Cubic.Bivariate Interpolation: Lagrange's Bivariate Interpolation, Newton's Bivariate Interpolation. Least square approximation.

Numerical differentiation: Numerical differentiation based on Interpolation, Numerical differentiation based on finite differences (forward, backward and central), Numerical Partial differentiation.

Unit III. Numerical Integration (11 L)

Numerical Integration based on Interpolation. Newton-Cotes Methods, Trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule. Determination of error term for all above methods. Convergence of numerical integration: Necessary and sufficient condition (with proof). Composite integration methods; Trapezoidal rule, Simpson's rule.

Unit IV. Ordinary Differential Equations (IVP) (10 L)

Single step methods: Taylor`s series method, Picard`s method of successive approximations, Euler`s Method (with geometrical interpretation and error analysis), Modified Euler`s method(with geometrical interpretation and error analysis), Runge-Kutta method of second and fourth order (with geometrical interpretation and error analysis)

Recommended Books
1. Kendall E. and Atkinson, An Introduction to Numerical Analysis, Wiley.
2. M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. S.D. Comte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGraw Hill International Book Company.
4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.
5. Hildebrand F.B., Introduction to Numerical Analysis, Dover Publication, NY.
6. Scarborough James B., Numerical Mathematical Analysis, Oxford University, Press, New Delhi.

Suggested Practicals :

1) Lagrange's Interpolation, Newton`s divided interpolation,
2) Gregory-Newton forward/backward difference Interpolation and Stirling Interpolation.
3) Hermite Interpoaltion, Piece-wise interpolation, Bivariate Interpolation, Least Square approximation
4) Numerical differentiation: Finite differences (forward, backward and central), Numerical Partial differentiation
5) Numerical Integration: Trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule. Composite integration methods: Trapezoidal rule, Simpson's rule.
6) Ordinary differential equations(IVP) by Taylor series method, Picard`s method, Euler`s method, Modified Euler`s method, Runge Kutta method.

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T.Y.B.Sc. Mathematics **Course: SMAT06AC**

Title: Number Theory and Projects

Learning Objectives:

1) To study Basics of Number Theory and Applications.
2) To learn LaTeX and SageMath.
3) To improve mathematical writing and presentation skills.

<u>UNIT I</u> Basics of Number Theory **(15 Lectures)**

Review of Divisibility, Primes and The fundamental theorem of Arithmetic. Congruences: Definition and elementary properties, Complete residue system modulo m; Reduced residue system modulo m; Euler's function and its properties, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem, Linear congruence, The Chinese remainder Theorem. The linear equations $ax + by = c$: The equations $x^2 + y^2 = p$ where p is a prime.The equation $x^2 + y^2 = z^2$; Pythagorean triples, primitive solutions, the equations $x^2 + y^2 = z^2$ and $x^2 + y^2 = z^4$ have no solutions (x; y; z) with xyz = 0: Every positive integer n can be expressed as sum of squares of four integers, introduction to Different Arithmetic functions.

UNIT II **Applications of Number Theory to Cryptography (15 Lectures)**

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Crypto-systems, symmetric key cryptography, simple examples such as shift cipher, Affine cipher, Hill's cipher, Vigenere cipher. Concept of Public Key Crypto-system; RSA Algorithm. An application of Primitive Roots to Cryptography.

Unit III **Introduction to a Computer Algebra Systems (SageMath) (15 Lectures)**

Introduction to SageMath.
Interface of LaTeX with SageMath.
Exploring concepts in Calculus, Number Theory, Group Theory using SageMath.

Unit IV **Applications of Linear Algebra (15 Lectures)**

In Singular Value Decomposition, Image Processing, Dimension Reduction, Stochastic Matrices, Forecasting Problems, Least Squares Method ,Constructing Curves and Surfaces Through Specified Points, Electrical Networks, Geometric Linear Programming, Cubic Spline Interpolation, Markov Chains, Graph Theory, Games of Strategy, Economic Models, Forest Management, Computer graphics, Fractals, Chaos, Cryptography,

References:

1. Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc.
2. David M. Burton, An Introduction to the Theory of Numbers, Tata McGraw Hill Edition.
3. "Group Theory — An Expedition with SageMath"Ajit Kumar and Vikas Bist to be published by Narosa Publishing House, New Delhi.
4. "Calculus using Sage" by Sang-Gu Lee, Robert Beezer, Ajit Kumar and othes published by Kyungmoon Books
5. Online book on "Linear Algebra Sang-Gu Lee, Ajit Kumar and others.
6. Mathematical Computation with Sage by Paul Zimmermann
7. A First Course in Linear Algebra by Robert Beezer available online
8. Abstract Algebra: Theory and Applications by Tom Judson and Robert Beezer
9. An Introduction to SAGE Programming: With Applications to SAGE Interacts for Numerical Methods by Razvan A Mezei
10. Elementary Linear Algebra – Howard Anton, Chris Rorres
11. Linear Algebra Miniatures – Jiri Matousek

Suggested Projects (To be done in Practicals):
(One reference book is mentioned for each topic but students can refer to others with the approval of the mentor.)

1. Quadratic Residues and Reciprocity.
   Tom M. Apostol, Introduction to Analytical number Theory, Springer International Student Edition.

2. Laplace transforms and Applications to ODE and PDE.
   Joel L. Schiff, The Laplace Transform Theory and Applications, Springer.

3. Group actions and Symmetry
   David S. Dummit, Richard M. Foote, Abstract Algebra, Third Edition

4. i) Study of Polya theory of Counting:
   ii) Hall's Marriage Theorem, Graph theory & Applications:
   K.H. Rosen, Discrete Mathematics and its Applications, Tata McGraw Hill Publishing Company, New Delhi,(Sixth edition).

5. Diophantine Equations.
   David Burton, Elementary Number Theory

6. i) Bessel Functions and the vibrating membrane:

   ii) Sturm-Liouville Boundary value problems, Eigenvalues, Eigenfunctions:

   G. F. Simmons, Differential Equations with Applications and Historical Notes, McGRAW-Hill International.

7. Algorithms in Cryptography:
   a) Kenneth H. Rosen, Discrete Mathematics and Its Applications, 7th Edition, McGraw Hill, 2012.
   b) Douglas R. Stinson, Cryptography Theory and Practice, 3rd Edition 2005.

8. Applications of Linear Algebra in Machine Learning Algorithms.

Evaluation System:

CIA I - 20 marks Test
CIA II - 20 marks Assignment/test
End Semester Exam – 60 marks project
Project will be assessed by 3 experts (2 internal + 1 external). Internal expert will give marks out of 20 and external experts will give marks out of 40. Project Report has to be typed in LaTeX.
Practical Exam:
Students have to do Project Presentation as follows:
1. **Progress report** for 20 marks in the month of January (**Practical CIA**).
2. **Final project presentation** for 30 marks during end semester exam. Presentation should be done using Beamer.
   (**End Semester Practical exam**). It will be evaluated by 2 external experts.

Eligibility Criteria for admission to this applied component:
Student should have studied Mathematics till the 4th semester.
