St. Xavier's College (Autonomous), Mumbai

Syllabus of the courses offered by the Department of Mathematics

(2015-16)

St. Xavier's College – Autonomous Mumbai

Syllabus
For ODD Semester Courses in
MATHEMATICS
(2015-2016)

Contents:

Theory Syllabus for Courses:

F.Y. B.Sc. **Course: S.MAT.1.01**
Title: CALCULUS AND ANALYTIC GEOMETRY I

Learning Objectives:
1. Intermediate value theorem and Mean Value Theorems.
2. Curve Tracing.

Number of lectures: 60

UNIT 1

Limit and continuity of functions of one variable **(20 lectures)**

a)Absolute value of a real number and the properties .
b)Intervals in R, neighbourhoods and deleted neighbourhoods of a real number, bounded subsets of R. l.u.b. and g.l.b. of sets and examples. c) Graphs of functions. Graph of a bijective function and its inverse.
d)Statement of rules for finding limits, sum rule, difference rule, product rule, constant multiple rule, quotient rule. (with proofs)
e)Sandwich theorem of limits (with proof)
f) Limit of composite functions (with proof)
g) Continuity of a real valued function in terms of limits and definition of continuity and two sided limits
h) Graphical representation of continuity of a real valued function
i) Removable discontinuity at a point of a real valued function and extension of a function having removable discontinuity at a point to a function continuous at that point. j) Continuity of polynomials and rational functions
k) Continuity of a real valued function over an interval.
l) Intermediate value property
m) A continuous function on a closed and bounded interval is bounded and attains its bounds

UNIT 2

Differentiability of functions of one variable **(20 lectures)**

a) Definition of derivative of a real valued function at a point, notion of differentiability, geometric interpretation of a derivative of a real valued function at a point, differentiability of a function over an interval, statement of rules of differentiability, chain rule of finding derivative of composite differentiable functions, derivative of inverse function (without proof).
b) Differentiable functions are continuous, but the converse is not true. Higher order derivatives, examples of functions which are differentiable n times but not (n + 1) times. Leibnitz Theorem for nth order derivative of product of two n times differentiable functions.

UNIT 3

Applications of derivatives **(20lectures)**

a)Mean Value Theorems : Rolle's Mean Value Theorem, Lagrange's Mean Value Theorem, Cauchy's Mean Value Theorem.

b) L' Hospitals rules, Taylor's polynomial and Taylor's Theorem with Lagrange's form of remainder.

c)Extreme values of functions, absolute and local extreme critical points, increasing and decreasing functions, the second derivative test for extreme values.

d) Graphing of functions using first and second derivatives, the second derivative test for concavity, points of inflection

e) Limits as x approaches infinity . Asymptotes – horizontal and vertical.

List Of Recommended Reference Books

1. Calculus and Analytical Geometry, G. B. THOMAS and P. L. FINNEY, Ninth Edition Addison – Wesley, 1998.

2. A Course in Calculus and Real Analysis, SUDHIR R. GHORPADE and BALMOHAN V. LIMAYE, Springer International Edition.

F.Y. B.Sc. **Course: S.MAT.1.02**

Title: DISCRETE MATHEMATICS I

Learning Objectives:

1. To learn about Euclidian algorithm
2. Properties of Congruences
3. Pigeon hole principle

Number of lectures: 60

UNIT 1

Integers and divisibility **(20 lectures)**

a) Statements of well-ordering property of non-negative integers

b) Principle of finite induction (first and second) as a consequence of well-ordering property

c) Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.d.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a and b and that the g.c.d. can be expressed as ma + nb integers.

d) Primes, Euclid's lemma, Fundamental Theorem of Arithmetic

UNIT 2

Functions and Counting Principles **(20 lectures)**

a) Review of functions , Injective, surjective, bijective functions. Composite of injective, surjective, bijective functions are injective, surjective and bijective respectively

b) Invertible functions ,Examples of functions including constant, identity, projection, inclusion.

c) Binary operation as a function, properties, examples. Definition and examples.

d) Pigeon Hole Principle and its applications .

e) Finite and infinite sets, cardinality of a finite set

f) The number of subsets of a finite set having n elements is 2^n

g) Equivalent sets, countable, uncountable sets.

h) A set is not equivalent to its power set.

UNIT 3

Integers and Congruences **(20lectures)** a) The set of primes is infinite.

b) Congruences: Definition and elementary properties.

c) Euler function, invertible elements modulo n

d) Euler's Theorem, Fermat's Little Theorem, Wilson's Theorem(without proof).

e) Applications: Solution of linear congruences

List Of Recommended Reference Books

1. Elementary Number Theory, DAVID M. BURTON, Second Edition, BS, New Delhi.
2. Discrete Mathematics, NORMAN L. BIGGS, Revised Edition, Clarendon Press, Oxford 1989.

S.Y. B.Sc. -Mathematics **Course: S.Mat.3.01**

Title: Calculus and Analysis

Learning Objectives:

To learn about (i) lub axiom of R and its consequences

(ii) Convergence of sequences

(iii) Convergence of infinite series

Number of lectures : 45

UNIT 1

Real Numbers : **(15 lectures)**

(a) Statements of algebraic and order properties of R.

(i) Elementary consequences of these properties including the A.M. - G.M. inequality, Cauchy-Schwarz inequality and Bernoulli inequality (without proof). (b) (i) Review of absolute value and neighbourhood of a real number.

(ii) Hausdorff property.

(c) Supremum (lub) and infimum (glb) of a subset of R, lub axiom of R. Consequences of lub axiom of R including (i) Archimedian property.

(ii) Density of rational numbers.

(iii) Existence of n^{th} root of a positive real number (in particular square root).

(iv) Decimal representation of a real number.

(d) (i) Nested Interval Theorem.

(ii) Open sets in R and closed sets as complements of open sets.

(iii) Limit points of a susbset of R, examples, characterisation of a closed set as a set containing all its limit points.

(e) Open cover of a susbset of R, Compact susbset of R, Definition and examples.
(Prove that a closed and bounded interval [a, b] is compact)

UNIT 2

Sequences, Limits and Continuity : (15 lectures)

(a) Sequence of real numbers, Definition and examples. Sum, difference, product, quotient and scalar multiple of sequences.

(b) Limit of a sequence, Convergent and divergent sequences, Uniqueness of limit of a convergent sequence, Algebra of convergent sequences, Sandwich theorem of sequences. Limits of standard sequences such as $\left\{\frac{1}{n^a}\right\}$ where a>0, $\{a^n\}$ where $|a| < 1$, $\left\{n^{\frac{1}{n}}\right\}, \left\{a^{\frac{1}{n}}\right\}$ where a > 0, {1/ n!}, $\left\{\frac{a^n}{n!}\right\}$ where a∈R .
Examples of divergent sequences.

(c) (i) Bounded sequences. A convergent sequence is bounded.
(ii) Monotone sequences, Convergence of bounded monotone sequences. The number e as a limit of a sequence, Calculation of square root of a positive real number.

(d) (i) Subsequences.
(ii) Limit inferior and limit superior of a sequence.
(iii) Bolzano-Weierstrass theorem of sequences.
(iv) Sequential characterisation of limit points of a set.

(e) Cauchy sequences, Cauchy completeness of R.

(f) Limit of a real valued function at a point :- (i) Review of the ε − δ definition of limit of functions at a point, uniqueness of limits of a function at a point whenever it exists.
(ii) Sequential characterization for limits of functions at a point, Theorems of limits (Limits of sum, difference, product, quotient, scalar multiple and sandwich theorem)
(iii) Continuity of function at a point, ε−δ definition, sequential criterion, Theorems about continuity of sum , difference , product , quotient and scalar multiple of functions at a point in the domain using ε − δ definition or sequential criterion. Continuity of composite functions. Examples of limits and continuity of a function at a point using sequential criterion.
(iv) A continuous function on closed and bounded interval is bounded and attains bounds.

UNIT 3

Infinite Series : (15 lectures)

(a) Infinite series of real numbers, the sequence of partial sums of an infinite series, convergence and divergence of series, sum, difference and multiple of convergent series are again convergent.

(b) Cauchy criterion of convergence of series. Absolute convergence of series, Geometric series. (c) Alternating series, Leibnitz' Theorem, Conditional convergence. An absolutely convergent series is conditionally convergent, but the converse is not true.

(d) Rearrangement of series (without proof). Cauchy condensation test (statementonly), application to convergence of p- series (p > 1). Divergence of Harmonic series (e) Tests for absolute convergence, Comparison test, Ratio test, Root test.

(f) Power series, Radius of convergence of power series:- The exponential, sine and cosine series.

(g) Fourier series, Computing Fourier Coefficients of simple functions such as x, x^2 , |x|, piecewise continuous functions on $[-\pi, \pi]$.

List of Recommended Reference Books

1. Robert G. Bartle and Donald R. Sherbet : Introduction to Real Analysis, Springer Verlag.
2. R. Courant and F. John : Introduction to Calculus and Analysis Vol I, Reprint of First Edition, Springer Verlag, New York 1999.
3. R. R. Goldberg: Methods of Real Analysis, Oxford and IBH Publication Company, New Delhi.
4. T. Apostol: Calculus Vol I, Second Edition, John Wiley.
5. M. H. Protter: Basic elements of Real Analysis, Springer Verlag, New York 1998.
6. Howard Anton, Calculus - A new Horizon, Sixth Edition, John Wiley and Sons Inc,1999.
7. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.

S.Y. B.Sc. -Mathematics **Course: S.Mat.3.02**
Title: Linear Algebra

Learning Objectives:
To solve the system of equations using row echelon form of a matrix, To study the structure of a vector space through its basis and to understand Gram-Schmidth orthogonalisation process in an inner product space.

Number of lectures : 45

UNIT 1

System of linear equations and matrices : (15 lectures)

(a) System of homogeneous and non-homogeneous linear equations. The solution of system of m homogeneous equations in n unknowns by elimination and their geometric interpretation for (m,n) = (1,2),(1,3),(2,2),(2,3),(3,3).

Definition of n tuples of real nos., sum of two n tuples and scalar multiple of n tuple. The existence of non-trivial solution of such a system for m<n. The sum of two solutions and a scalar multiple of a solution of such a system is again a solution of the system.

(b) Matrices over R, the matrix representation of system of homogeneous and non-homogeneous linear equations. Addition, scalar multiplication and multiplication of matrices, transpose of a matrix. Types of matrices. Transpose of product of matrices, invertible matrices, product of invertible matrices.

(c) Elementary row operations on matrices, row echelon form of a matrix, Guassian elimination method. Application of Guass elimination method to solve system of linear equations. Row operations and elementary matrices, elementary matrices are invertible and invertible matrix is a product of elementary matrices.

UNIT 2

Vector spaces over R : (15 lectures)

(a) Definition of vector space over R. Ex. such as: Euclidean space R^n, space of R^∞ of sequences over R, space of m × n matrices over R, the space of polynomials with real co-efficients, space of real valued functions on a nonempty set.

(b) Subspaces- definition and examples including: lines in R^2, lines and planes in R^3, solutions of homogeneous system of linear equations, hyperplane, space of convergent sequences, space of symmetric and skew symmetric, upper triangular, lower triangular, diagonal matrices, and so on.

(c) Sum and intersection of subspaces, direct sum of vector spaces, linear combination of vectors, convex sets, linear span of a subset of a vector space, linear dependence and independence of a set.

(d) (For finitely generated vector spaces only) Basis of a vector space , basis as maximal linearly independent set and as a minimal set of generators, dimension of a vector space.

(e) Row space, column space of an m × n matrix over and row rank, column rank of a matrix. Equivalence of row rank and column rank, computing rank of a matrix by row reduction.

UNIT 3

Inner product spaces : (15 lectures)

(a) Dot product in R^n, definition of general inner prduct on a vector space over R. Ex. such as $C[-\pi, \pi]$ and so on.

(b) Norm of a vector in an inner product space, Cauchy-schwartz inequality, triangle inequality. Orthogonality of vectors, Pythagoras theorem and geometric application in R^2, projection of a line, projection being the closest approximation.

Orthogonal complements of a subspace, orthogonal complement in R^2 and R^3.

Orthogonal sets and orthonormal sets in an inner product space.

Orthogonal and orthonormal bases. Gram-Schmidth orthogonalisation process, ex. in R^2 , R^3 and R^4.

List of Recommended Reference Books

1. Introduction to linear algebra by Serge Lang, Springer verlag.
2. Linear algebra- a geometric approach by Kumaresan, Prentice-hall of India private limited, New Delhi.
3. Linear algebra by Hoffman and Kunze, Tata McGraw-Hill, New Delhi.

S.Y. B.Sc. -Mathematics Course: S.Mat.3.03

Title: Computational Mathematics

Learning Objectives:

Introduction to Algorithms and Graph Theory

Number of lectures : 45

UNIT 1

Algorithms : **(15 lectures)**

(a) Definition of an algorithm, characteristics of an algorithm.Selection and iterative constructs in pseudo code, simple examples such as
   - (i) Finding the number of positive and negative integers in a given set.
   - (ii) Finding absolute value of a real number.
   - (iii) Exchanging values of variables.
   - (iv) Sum of n given numbers. Sum of a series.

(b) Algorithms on integers:
   - (i) Computing quotient and remainder in division algorithm.
   - (ii) Converting decimal number to a binary number.
   - (iii) Checking if a given number is a prime. Finding Pythagorean triples .
   - (iv)Euclidean algorithm to find the g. c. d of two non-zero integers.
   - (v) To test whether a number is a prime. To find first 100 primes.

UNIT 2 **(15 lectures)**

Use of arrays

(a) Searching and sorting algorithms, including
   - (i) Finding maximum and/or minimum element in a finite sequence of integers.
   - (ii) The linear search and binary search algorithms of an integer x in a finite sequence of distinct integers.
   - (iii) Sorting of a finite sequence of integers in ascending order. Bubble sort
   - (iv) Merging of two sorted arrays.

(b) Algorithms on matrices:
   - (i) Addition and multiplication of matrices.
   - (ii) Transpose of a matrix.
   - (iii) Row sum and column sum of a matrix.

(c) Recursion, Examples including:
   - (i) Fibonacci sequence
   - (ii) Computing a_n for non-negative integer n.
   - (iii) Euclidean algorithm.
   - (iv) Searching algorithm
   - (v) Factorial of a non-negative integer.

UNIT 3

Graphs : **(15 lectures)**

(a) Introduction to graphs: Types of graphs: Simple graph, Multigraph, psuedograph, directed graph, directed multigraph. One example/graph model of each type to be discussed.

(b) (i) Graph Terminology: Adjacent vertices, degree of a vertex, isoloated vertex, pendant vertex in a undirected graph.

(ii) The handshaking Theorem for an undirected graph. An undirected graph has an even number odd vertices.

(c) Some special simple graphs: Complete graph, cycle, wheel in a graph, Bipartite graph, regular graph.

(d) Representing graphs and graph isomorphism. (i) Adjacency matrix of a simple graph.

(ii) Incidence matrix of an undirected graph.

(iii) Isomorphism of simple graphs.

(e) Connectivity:-

(i) Paths, circuit (or cycle) in a graph.

(ii) Connected graphs, connected components in an undirected graph, A strongly connected directed graph, A weakly connected directed graph. A cut vertex.

(iii) Connecting paths between vertices.

(iv) Paths and isomorphisms.

(v) Euler paths and circuits, Hamilton paths and circuits. Dirac's Theorem, Ore's Theorem

(vi) Shortest path problem, The shortest path algorithm - Dijkstra's Algorithm.

(f) Trees :-

(i) Trees: Definition and Examples.

(ii) Forests, Rooted trees, subtrees, binary trees.

(iii) Trees as models.

(iv) Properties of Trees.

(g) Application of Trees:

(i) Binary Search Trees, Locating and adding items to a Binary Search Tree.

(ii) Decision Trees (simple examples).

(iii) Game Trees, Minimax strategy and the value of a vertex in a Game Tree. Examples of games such as Nim and Tic-tac-toe.

(h) Spanning Tree

(i) Spanning Tree, Depth-First Search and Breadth-First Search.

(ii) Minimum Spanning Trees, Prim's Algorithm, Kruskal's Algorithm.

(The Proofs of the results in this unit are not required and may be omitted)

List of Recommended Reference Books

1. Kenneth H. Rosen : Discrete Mathematics and Its Applications, McGraw Hill Edition.

2. Bernard Kolman, Robert Busby, Sharon Ross: Discrete Mathematical Structures, Prentice-Hall India.
3. Norman Biggs: Discrete Mathematics, Oxford.
4. Douglas B. West: Introduction to graph Theory, Pearson.
5. Frank Harary, Graph Theory, Narosa Publication.
6. R.G. Dromey, How to Solve it by computers, Prentice-Hall India.
7. Graham, Knuth and Patashnik: Concrete Mathematics, Pearson Education Asia Low Price Edition.
8. Thomas H. Cormen, Charles E. Leisenon and Ronald L. Rivest: Introduction to Algorithms, Prentice Hall of India, New Delhi, 1998 Edition.

Practical: **Course:S.Mat.3. PR**
Computational Mathematics

(1) Algorithms on integers and prime numbers
(2) Algorithms on one dimensional arrays.
(3) Algorithms on two dimensional arrays and matrices.
(4) While loop, G.C.D. etc.
(5) (i) Drawing a graph, Checking if a degree sequence is graphical.
(6) (ii) Representing a given graph by an adjacency matrix and drawing a graph having given matrix as adjacency matrix.
(7) Determining whether the given pairs of graphs are isomorphic.
(Exhibiting an isomorphism between the isomorphic graphs or proving that none exists) (8) To determine whether the given graph is a tree. Construction of Binary Search Tree and applications to sorting and searching.
Finding Spanning Tree using Breadth First Search and/or Depth First search

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T.Y. B.Sc. Maths **Course: S.Mat. 5.01**
Title: REAL ANALYSIS and MULTIVARIABLE CALCULUS

Learning Objectives:
1. To understand Riemann Integrability of bounded functions .
2. first and second Fundamental Theorem of Calculus and fubini's theorem of rectangles

Number of lectures: 45

UNIT 1

Riemann Integration, Double and Triple Integrals (12 lectures)

(a) Uniform continuity of a real valued function on a subset of R

(i) Definition.

(ii) a continuous function on a closed and bounded interval is uniformly continous (only statement).

(b) Riemann Integration.

(i) Partition of a closed and bounded interval [a; b], Upper sums and Lower sums of a bounded real valued function on [a; b]. Refinement of a partition, Definition of Riemann integrability of a function. A necessary and sufficient condition for a bounded function on [a; b] to be Riemann integrable.(Riemann's Criterion)

(ii) A monotone function on [a; b] is Riemann integrable.

(iii) A continuous function on [a; b] is Riemann integrable.

A function with only finitely many discontinuities on [a; b] is Riemann integrable.

Examples of a Riemann integrable function which is discontinuous at all rational numbers.

(c) Algebraic and order properties of Riemann integrable functions. (i) Riemann Integrability of sums, scalar multiples and products of integrable functions. The formulae for integrals of sums and scalar multiples of Riemann integrable functions.

(ii) If f is Riemann integrable on [a; b], and a < c < b, then f is Riemann integrable on [a; c] and [c; b]

Unit 2 (11 lectures)

(a) First and second Fundamental Theorem of Calculus.

(b) Integration by parts and change of variables formula.

(c) Mean Value Theorem for integrals.

(d) The integral as a limit of a sum, examples.

(e) Double and Triple Integrals

(i) The definition of the Double (respectively Triple) integral of a bounded function on a rectangle (respectively box).

(ii) Fubini's theorem over rectangles.

(iii) Properties of Double and Triple Integrals:

(1) Integrability of sums, scalar multiples, products of integrable functions, and formulae for integrals of sums and scalar multiples of integrable functions.

(2) Domain additivity of the integrals.

(3) Integrability of continuous functions and functions having only finitely (countably) many discontinuities.

(4) Double and triple integrals over bounded domains.

(5) Change of variables formula for double and triple integrals (statement only).

UNIT 3

Sequences and series of functions: (11 lectures)

(a) Pointwise and uniform convergence of sequences and series of real valued functions. Weierstrass M-test. Examples.

(b) Continuity of the uniform limit (resp: uniform sum) of a sequence (resp: series) of real-valued functions. The integral and the derivative of the uniform limit (resp: uniform sum) of a sequence (resp: series) of real-valued functions on a closed and bounded interval. Examples.

UNIT 4 **(11 lectures)**

(a) Power series in R. Radius of convergence. Region of convergence. Uniform convergence. Term-by-term differentiation and integration of power series. Examples.

(b) Taylor and Maclaurin series. Classical functions defined by power series: exponential, trigonometric, logarithmic and hyperbolic functions, and the basic properties of these functions.

List Of Recommended Reference Books

1. Real Analysis Bartle and Sherbet.
2. Calculus, Vol. 2: T. Apostol, John Wiley.
3. Richard G. Goldberg, Methods of Real Analysis, Oxford & IBHPublishing Co. Pvt. Ltd., New Delhi.

Practical:

I) Riemann Integration.
II) Fundamental Theorem of Calculus.
III) Double and Triple Integrals; Fubini's theorem, Change of VariablesFormula.
IV) Pointwise and uniform convergence of sequences and series of functions.
V) Illustrations of continuity, differentiability, and integrability for pointwise and uniform convergence.
VI) Power series in R. Term by term differentiation and integration.
VII) Miscellaneous Theoretical questions based on Unit 1.
VIII) Miscellaneous Theoretical questions based on Unit 2.

T.Y. B.Sc. Maths Course: S.Mat. 5.02

Title: ALGEBRA

Learning Objectives:

1. To understand Cyclic groups, Lagrange's theorem and Group homomorphisms and isomorphisms.

2. To understand Normal groups.

Number of lectures: 45

UNIT 1

Groups and subgroups **(12 lectures)**

(a) Definition and properties of a group. Abelian group. Order of a group, finite and infinite groups. Examples of groups including (i) Z, Q, R, C under addition.

(ii) Q*, R* under multiplication.

(iii) Z_n, the set of residue classes modulo n under addition.

(iv) U(n), the group of prime residue classes modulo n under multiplication.

(v) The symmetric group S_n.

(vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetrices of a regular polygon of n sides.

(vii) Quaternion group.

(viii) Matrix groups $M_n(R)$ under addition of matrices, $GL_n(R)$, the set of invertible real matrices, under multiplication of matrices.

(b) Subgroups

Subgroups of $GL_n(R)$ such as $SL_n(R)$, $O_n(R)$, $SO_n(R)$, $SO_2(R)$ as group of 2 x 2 real matrices representing rotations, subgroup of n-th roots of unity.

Unit 2 **(11 lectures)**

(a)(i) Cyclic groups (examples of Z, Z_n) and cyclic subgroups.

(ii) Groups generated by a finite set, generators and relations.

Examples such as Klein's four group V_4, Dihedral group, Quaternion group.

(iii) The Center Z(G) of a group G, and the normalizer of an element of G as a subgroup of G.

(iv) Cosets, Lagrange's theorem.

(b) Group homomorphisms and isomorphisms. Examples and properties. Automorphisms of a group, inner automorphisms.

UNIT 3

Normal subgroups: (11 lectures)

(a) (i) Normal subgroups of a group. Definition and exmaples including center of a group. (ii) Quotient group.

(iii) Alternating group A_n, cycles. Listing normal subgroups of A_4, S_3.

(b) Isomorphisms theorems. (i) First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups). (ii) Second Isomorphism theorem. (iii) Third Isomorphism theorem.

Unit 4

(a) Cayley's theorem. **(11 lectures)**

(b) External direct product of a group. Properties of external direct products. Order of an element in a direct product, criterion for direct product to be cyclic. The groups Z_n and U(n) as external direct product of groups.

(c) Classification of groups of order 7.

List Of Recommended Reference Books

1. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second
2. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. W.B. Fraleigh, A first course in Abstract Algebra, Third edition,Narosa, New Delhi.
5. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.
6. P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Practical:

I) Groups Definitions and properties.
II) Subgroups, Lagrange's Theorem and Cyclic groups..
III) Groups of Symmetry and the Symmetric group S_n.
IV) Group homomorphisms, isomorphisms.
V) Normal subgroups and quotient groups.
VI) Cayley's Theorem and external direct product of groups.
VII) Miscellaneous Theoretical questions based on Unit 1 and 2.
VIII) Miscellaneous Theoretical questions based on Unit 3 and 4.

T.Y. B.Sc. Maths — Course: S.Mat.5.03

Title: Topology of metric spaces.

Learning Objectives:

1. Inrtroduction to Metric Spaces.

Number of lectures: 45

UNIT 1

Metric spaces **(12 lectures)**

(a) (i) Metrics spaces: Definition, Examples, including R with usual distance, discrete metric space.

(ii) Normed linear spaces: Definition, the distance (metric) induced by the norm, translation invariance of the metric induced by the norm. Examples including

(1) $\mathbb{R}^n$ with sum norm $|| \; ||_1$, the Euclidean norm $|| \; ||_2$, and the sup norm $|| \; ||_\infty$.

(2) $C[a,b]$, the space of continuous real valued functions on $[a,b]$ with norms $|| \; ||_1$, $|| \; ||_2$, $|| \; ||_\infty$, where $||f||_1 = \int_a^b |f(t)|dt$, $||f||_2 = \left(\int_a^b |f(t)|^2 dt\right)^{\frac{1}{2}}$, $||f||_\infty = \sup\{|f(t)|, t \in [a,b]\}$.

(3) $\ell_1, \ell_2, \ell_\infty$, the spaces of real sequences with norms $|| \; ||_1$, $|| \; ||_2$, $|| \; |_\infty$, where $||x||_1 = \sum_{n=1}^\infty |x_n|$, $||x||_2 = (\sum_{n=1}^\infty |x_n|^2)^{\frac{1}{2}}$, $||x||_\infty = \sup\{|x_n|, n \in \mathbb{N}\}$, for $x = (x_n)$.

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(iii) Subspaces, product of two metric spaces.

(b) (i) Open ball and open set in a metric space (normed linear space) and subspace Hausdorff property. Interior of a set.

(ii) Structure of an open set in R, namely any open set is a union of a countable family of pairwise disjoint intervals.

(iii) Equivalent metrics, equivalent norms.

(c) (i) Closed set in a metric space (as complement of an open set), limit point of a set (A point which has a non-empty intersection with each deleted neighbourhood of the point), isolated point. A closed set contains all its limit points. (ii) Closed balls, closure of a set, boundary of a set in a metric space.

UNIT 2 (11 lectures)

(a)(i) Distance of a point from a set, distance between two sets, diameter of a set in a metric space.

(ii) Dense subsets in a metric space. Separability, Rn is separable.

(b) (i) Sequences in a metric space.

(ii) The characterization of limit points and closure points in terms of sequences.

(iii)Cauchy sequences and complete metric spaces. Rn with Euclidean metric is a complete metric space. (c) Cantor's Intersection Theorem.

UNIT 3

Continuity: (11lectures)

(a) Definition of continuity at a point of a function from one metric space to another.

(i) Characterization of continuity at a point in terms of sequences, open sets.

(ii) Continuity of a function on a metric space. Characterization in terms of inverse image of open sets and closed sets.

UNIT 4 *(11 lectures)*

(iii) Urysohn's lemma.

(iv) Uniform continuity in a metric space, definition and examples (emphasis on R), open maps, closed maps.

List Of Recommended Reference Books

1. S. Kumaresan, Topology of Metric spaces.
2. W. Rudin, Principles of Mathematical Analysis.
3. R.G. Goldberg Methods of Real Analysis, Oxford and IBH Publishing House, NewDelhi.

4. P.K. Jain, K. Ahmed. Metric spaces. Narosa, New Delhi, 1996.
5. G.F. Simmons. Introduction to Topology and Modern Analysis. McGraw Hill, New York, 1963.

Practical:

I) Metric spaces and normed linear spaces. Examples.
II) Open balls, open sets in metric spaces, subspaces and normed linear spaces.
III) Limit points: (Limit points and closure points, closed balls, closed sets, closure of a set, boundary of a set, distance between two sets).
IV) Sequences
V) Continuity.
VI) Uniform continuity in a metric space.
VII) Miscellaneous Theoretical Questions based on Unit 1 and 2
VIII) Miscellaneous Theoretical Questions based on Unit 3 and 4

T.Y. B.Sc. Maths Course: S.Mat.5.04 Title: Numerical Methods

Learning Objectives:

1. Newton – Raphson method Chebyshev method etc & their rate of convergence
2. Different types of interpolation methods

Number of lectures: 45

UNIT 1

Transcendental and Polynomial equations **(12 lectures)**

(a) Iteration methods based on first and second degree equation
- (i) The Newton – Raphson method
- (ii) Secant method
- (iii) Muller method
- (iv) Chebyshev method
- (v) Multi-point iteration metbod

(b) Rate of convergence and error analysis
- (i) Secant method
- (ii) The Newton – Raphson method
- (iii) Methods of multiple roots

UNIT 2 **(11 lectures)**

(a) Polynomial equations
- (i) Birge-Vieta method
- (ii) Bairstow method

(iii)Graeffe's Method

UNIT 3

Interpolation and approximation **(11 lectures)**

(a) Higher order interpolation
(b) Finite difference operators and fundamental theorem of difference calculus
(c) Interpolating polynomial using finite differences, factorial notation (d) Hermite interpolation

UNIT 4 (11 lectures)

(a)Piecewise and spline interpolation
(b)Bivariate interpolation – Lagrange bivariate interpolation, Newton`s bivariate interpolation for equispaced points
(c)Least square approximation

List Of Recommended Reference Books

1. M.K.Jain , S.R.K. Iyengar and R.K.Jain. Numerical methods for Scientific and Engineering Computation, New age International publishers,Fourth Edition, 2003
2. S.D.Comte and Carl de Boor, Elementary Numerical analysis- An algorithmic approach, 3rd edition., McGraw Hill, International Book Company,1980.
3. F.B.Hildebrand, Introduction to Numerical Analysis, McGraw Hill, New York, 1956.

Practical:

I) Iteration methods based on first degree equation-Newton Raphson method , Secant method.
II) Iteration methods based on second degree equation-Muller method , Chebyshev method, Multi-point iteration metbod
III) Polynomial equations
IV) Higher order Interpolation/finite difference operators
V) Interpolating polynomial using finite differences / Hermite interpolation VI) Piecewise and spline interpolation / Bivariate interpolation

VII) Miscellaneous Theoretical questions based on Unit 1and 2
VIII)Miscellaneous Theoretical questions based on Unit 3 and 4

T.Y. B.Sc. Maths **Course: S.Mat.5.AC.01**

Title: COMPUTER PROGRAMMING AND SYSTEM ANALYSIS (JAVA PROGRAMMING & SSAD)

Learning Objectives:

1. To learn about OOP through java programming
2. Intro. to DBMS & RDBMS, SQL Commands & Functions, and C-languag

UNIT 1

Java Programming (20 lectures)

Introduction to JAVA Programming

What is java, history of java, different types of java programmes, java virtual machine, JDK tool.

Object oriented programming

Object oriented approach, Object oriented programming, objects and classes, behavior and attributes, fundamental principles of OOPs (encapsulation, inheritance – polymorphism. data abstraction).

Java Basics (Data Concepts)

Variables and data types, declaration variables, literals. numeric literals, Boolean literal, character literals, string literals, keywords, type conversion and casting ,shift operators.

Java Operators

Assignment operator, arithmetic operators ,relational operators, logical operators, bitwise operators , incrementing and decrementing operators , conditional operator, precedence and order of evaluation, statement and expressions

Exception handling

Command line arguments, Parsing , try – catch blocks , types of exception & how to handle them.

Loops and Controls

Control statement for decisions making: selection statements (if statement, if- else statement, if- else - if statement, switch statement), goto statement ,looping (while loop and do while loop and for loop), nested loops, breaking out of loops(break and continue statements), return statement.

Introduction to Classes and Methods

Defining classes, creating- instance and class variables, creating objects of a class, accessing instance variables of a class, Creating methods, naming methods, accessing methods of class, constructor methods, overloading methods.

UNIT 2

Structured System Analysis and Design: (05 lectures)

What is a system, characteristics system, types of information system –
Transaction Processing System (TPS), Management Information System (MIS), Decision Support System (DSS).

System Development Strategies

System Development Life Cycle (SDLC) method. Structured analysis development method. Element of structured analysis – Data Flow Diagrams (DFD), data dictionary.

Tools for determining System Requirements

What is requirement determination. fact finding techniques tools for documenting procedures and decisions – decision tree, decision table.

List Of Recommended Reference Books

1. Analysis and Design of Information System – James A. Senn (McGraw – Hill International Editions)------ (Chapters –1 & 3)

2. The complete reference - Java 2 :- Herbert schildt (TMH). (Chapters 1 to 7,10)

Practical:

Java programs that illustrate

I) the different types of operators
II) the concept of casting and shift operators.
III) the concept of selection statements.
IV) the concept of looping , nested loops, jumping statements
V) the concept of command line arguments ,parsing and try – catch blocks(exception handling)
VI) the concept of java class.
VII) the concept of java class that includes constructor with and without parameters.
VIII) the concept of java class that includes overloading methods

UNIT 3

SQL Commands and Functions **(16 lectures)**

Handling data

Selecting data using SELECT statement. FROM clause, WHERE clause, HAVING clause, ORDER BY, GROUP BY, DISTINCT and ALL predicates. Adding data with INSERT statement. Changing data with UPDATE statement. Removing data with DELETE statement.

Joining Tables

Inner joins, outer joins, cross joins, union.

Functions
Aggregate functions-AVG, SUM, MIN, MAX and COUNT. Date functions - DATEADD(), DATEDIFF(), GETDATE(), DATENAME(), YEAR, MONTH, WEEK, DAY. String functions - LOWER(), UPPER(), TRIM(), RTRIM(), PATINDEX(), REPLICATE(), REVERSE(), RIGHT(), SPACE().
Creating and Altering tables
CREATE statement, ALTER statement, DROP statement.
Views
Simple views, complex views, creating and editing views.
Constraints
Types of constraints, KEY constraints, CHECK constraints, DEFAULT constraints, disabling constraints.
Indexes
Understanding indexes, creating and dropping indexes, maintaining indexes.

UNIT 4

Basics in C- Language : **(09 lectures)** Program Structure
Header and body, use of comments, construction of the program.
Data Concepts
Variables, constants, and data types, declaring variables.
Simple Input/Output Operations
Character strings: printf(), scanf(), single characters: getchar(), putchar() Operators
Assignment operators, compound assignment operators, arithmetic operators, relational operators, logical operators, increment and decrement operators, conditional operator, precedence and order of evaluation, statements and expressions.
Type conversions
Automatic and explicit type conversions.

List Of Recommended Reference Books

1. Professional SQL Server 2000 Programming - Rob Vieira, Wrox Press Ltd, Shroff Publishers & Distributors Pvt Ltd, NewDelhi.(Chapters 4-10).
2. SQL Server 2000 Black Book - Patrick Dalton & Paul Whitehead, Dreamtech Press.

Practical:

I) Single table queries using operators with select columns and restricting rows of output.

II) Supply queries using SELECT command.

III) Supply queries using SELECT with FROM, WHERE and HAVING clauses.

IV) Supply queries using SELECT with ORDER BY, GROUP BY, DISTINCT, ALL and queries along with different clauses.

V) Queries using aggregate functions, string functions, date functions.

VI) Creating, updating, altering and deleting tables and views.

VII) Creating tables with defaults, integrity constraints, referential integrity constraints and check constraints both at the column and table levels.

St. Xavier's College – Autonomous Mumbai

Syllabus For Even Semester Courses in MATHEMATICS (2015-2016)

Contents:

Theory Syllabus for Courses:

F.Y. B.Sc. -Mathematics **Course: S.Mat.2.01**

Title: Calculus and Analytic Geometry II

Learning Objectives:

To learn about (i) Quadric surfaces

(ii) Partial derivatives and directional derivatives

(iii) Extremization of functions in two variables

Number of lectures : 60

UNIT 1

Analytic Geometry in Euclidean spaces: **(20 Lectures)**

(a) Review of vectors in R^2 and R^3, component form of vectors, basic notions such as addition and scalar multiplication of vectors, dot product of vectors, orthogonal vectors, length (norm) of a vector, unit vector, distance between two vectors, cross product of vectors in R^3, scalar triple product (box product), vector projections.

(b) Lines and planes in space, equation of sphere, cylinders and quadric surfaces.

(c) Polar co-ordinates in R^2, polar graphing of curves like $r = a \sin\Theta$, $r = a \cos\Theta$, $r = a (1 - \sin\Theta)$.

(d) Relationship between polar and Cartesian co-ordinates in R^2, cylindrical and spherical co ordinates in R^3.Also relationships of these co-ordinates with cartesian co-ordinates and each other.

UNIT 2

Limits and continuity of functions of two and three variables: **(20 Lectures)**

(a) (i) Open disc in R^2 and R^3, boundary of open disc, closed disc in R^2 and R^3, bounded regions, unbounded regions in R2 and R3.

(ii)Real valued functions of two or three variables with examples. Level curves for functions of two variables. Use of level curves to draw graphs of $z = f(x, y)$, especially quadric surfaces. definition of a limit of a real valued function of two variables (only brief statement).

(b) Statement of rules of limits of functions in two (or three) variables:- Sum rule, difference rule, product rule, constant multiple rule, quotient rule, power rule. Apply these rules to determine limits of polynomial & rational functions. Definition of continuity of functions of two(or three) variables in terms of limits.

(c) Definition of a path. Limit of a function along paths. Two path test for non-existence of a limit with examples. Sandwich theorem for a function of two variables (without proof).

(d) Calculating limits by changing to polar co-ordinates (illustrate with examples).

(e) Vector valued functions of one and several variables, planar and space curves, component functions, vector fields, graphs of vector valued function like (cos t, sin t), (cos t, sin t, 1), (cos t, sint, t). Limits of vector valued function by taking limits of component functions.

UNIT 3

Differentiability of functions of two variables:- **(20 Lectures)**

(a) (i) Partial derivatives of a real valued functions in two variables, the relationship between continuity & existence of partial derivatives at a point. Second order partial derivatives.

Mixed derivative theorem for two variables (without proof). The increment theorem for two variables (without proof).

(ii) Differentiability of a function in two variables at a point over a disc, linearization of a differentiable function at a point.

(iii) Chain rule for composite functions in two &/or three variables (iv) Implicit differentiation.

(b) Directional derivatives in a plane, interpretation of directional derivatives, gradient vector, relation between directional derivatives and gradient.

(c) Geometric interpretation of partial derivatives and its relation to the tangent plane at a point. (d) Extreme values of a function in two variables. Local maximum, local minimum and first derivative test for local extreme values (without proof). Critical points, saddle points, second derivative test for local extreme values (without proof).

(e)The method of Lagrange's Multiplier to obtain extrema of a function in two or threevariables.

(f) Derivative of vector valued function as derivative of component functions. Statement of rules of differentiation – sum, difference, product, constant multiple. Chain rule for composite function s (without proof). Geometric interpretation of derivatives. Derivative of dot and cross products.

<u>List of Recommended Reference Books</u>

1. Calculus and Analytical Geometry, G. B. THOMAS and R. L. FINNEY, Ninth Edition, Addison-Wesley, 1998.
2. Howard Anton, Calculus - A new Horizon, Sixth Edition, John Wiley and Sons Inc,1999.
3. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.

F.Y. B.Sc. Mathematics **Course:** S.Mat.2.02

Title: DISCRETE MATHEMATICS II

Learning Objectives:

1. To learn about Transpositions, Partition of a set.
2. To learn about Fundamental theorem of Algebra.

Number of lectures: 60

<u>UNIT 1</u>

Principles of Counting and Equivalence Relations (20 lectures)

(a) Addition and multiplication Principles

(b) Counting sets of pairs

(c) Cartesian product of n sets

(d) Equivalence relation, Equivalence classes, Properties, Examples including the relation modulo n on Z.

(e) An equivalence relation induces partition of a set and a partition of a set defines an equivalence relation

UNIT 2

Principles of Counting and permutations (20 lectures)

(a)Distribution of objects, multinomial numbers, combinatorial interpretations, multinomial theorem

(b) Inclusion and Exclusion (Sieve, Principle)

(c)

(i) Number of functions from a finite set X to a finite set Y

(ii) Number of injective functions from a finite set X to a finite set Y, where |X| < |Y|

(iii) Number of subjective functions from a set X to a finite set Y, where |X| > |Y| (d) Permutations

(i) Permutations on n symbols. The set S n and the number of permutations in S n

(ii) Composition of two permutations as a binary operation in Sn composition on permutation is non-commutative if n = 3.

(iii) Cycles and transpositions, representation of a permutation as product of disjoint cycles. Listing permutations in S3, S4, etc.

(iv) Sign of a permutation, sign of transposition is -1, multiplicative property of sign, odd and even permutations, number of even permutations, number of even permutations in Sn is (v) Partition of a positive integer, its relation to decomposition of a permutation as product disjoint cycles, conjugate of a permutation

(e) Derangements on n symbols, d n ,the number of derangements of {1,2, …n}

(f) Groups, subgroups, cyclic groups, examples, cosets, lagranges theorem

UNIT 3

Polynomials (20 lectures)

(a) The set F [X] of polynomials in one variable over F where F = Q, R, or C

(b) Addition, Multiplication of two polynomials, degree of a polynomial, basic properties.

(c) Division algorithm in F[X] (with proof) and g.c.d. of two polynomials, and its basic properties (with proof), Euclidean algorithm, (without proof)

(d) Roots of a polynomial, multiplicity of a root, Factor theorem. A polynomial of degree n over F has at most n roots. Relation between roots of a polynomial and its coefficients.

Necessary condition for p/q to be a root of polynomial in Z[X]

(e) Statement of Fundamental Theorem of Algebra and its elementary consequences

(f) Complex number z is a root of a polynomial f(X) over R if and if its conjugatge z is a root of f (X).

(g) Polynomials in R[X] can be expressed as a product of linear and quadratic polynomial in C[X].

(h) Statement of De Morgan's Theorem, roots of unity, Primitive n^{th} roots of unity, n^{th} roots of a complex number

List Of Recommended Reference Books

(1) Discrete Mathematics, NORMAN L. BIGGS, Revised Edition, Clarendon Press, Oxford 1989

(2) Discrete Mathematics and its applications, KENNETH H. ROSEN, McGraw Hill International Edition

(3) Algebra, I. N Herstein, Algebra, Fraleigh, University

(4) Algebra, N.S. Gopalkrishnan.

SEMESTER IV **COURSE: S.MAT.4.01**

Calculus and Analysis [45 LECTURES]

Reference for Unit 1: Chapter 2, Sections 7, 8, 9, 10 and Chapter 3, Sections 14, 15, 16, 17, 18, 19, 20 of Differential Equations with Applications and Historical Notes, G.F. Simmons, McGraw Hill.

Unit 2. Multiple integrals (15 Lectures)

Review of functions of two and three variables, partial derivatives and gradient of two or three variables.

(a) Double integrals: (i) Definition of double integrals over rectangles. (ii) Properties of double integrals.

(iii) Double integrals over bounded regions. (b) Statement of Fubini's Theorem, Double integrals as volumes.

(c) Applications of Double integrals: Average value, Areas, Moments, Center of Mass.

(d) Double integrals in polar form.

(e) Triple integrals in Rectangular coordinates, Average, volumes.

(f) Applications of Triple integrals: Mass, Moments, Parallel axis Theorem.

(g) Triple integrals in Spherical and Cylindrical coordinates.

Reference for Unit 2: Chapter 13, Sections 13.1, 13.2, 13.3, 13.4, 13.5, 13.6 of Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, Ninth Edition, Addison-Wesley, 1998.

Unit 3. Integration of Vector Fields (15 Lectures) (a) Line Integrals, Definition, Evaluation for smooth curves. Mass and moments for coils, springs, thin rods.

(b) Vector fields, Gradient fields, Work done by a force over a curve in space, Evaluation of work integrals.

(c) Flow integrals and circulation around a curve. (d) Flux across a plane curve.

(e) Path independence of the line integral of F region, F being a vector field Conservative fields, potential function.

(f) The Fundamental theorems of line integrals (without proof).

(g) Flux density (divergence), Circulation density (curl) at a point.

(h) Green's Theorem in plane (without proof), Evaluation of line integrals using Green's Theorem.

Reference for Unit 3: Chapter 14 of 14.1, 14.2, 14.3, 14.4 Calculus and Analytic Geom- etry, G.B. Thomas and R. L. Finney, Ninth Edition, Addison-Wesley, 1998.

The proofs of the results mentioned in the syllabus to be covered unless indicated otherwise.

Recommended Books

1. G.B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley, 1998.
2. G.F. Simmons: Differential Equations with Applications and Historical Notes, McGraw Hill.
3. Sudhir Gorpade and Balmohan Limaye : A course in Multivariable calculus and Analysis, Springer.

Course: S.MAT.4.02

Linear Algebra

Learning Objectives: *To understand linear maps and isomorphism between vector spaces, determinant function as an n form on vector space Rn and eigen values and eigen vector of a linear map.*

Unit 1. Linear transformations. (15)

(a) Linear transformations- defn and properties and eg including Prjection from R_n to R_m, rotations and reflections, map The defined by matrix, orthogonal projection in R_n, functionals. (b) Sum and scalar multiple of a linear transformation. space L(U,V) of Linear transformation from U to V.
The dual space V.*
(c) Kernel and image of a linear transformation.
Rank nullity thm.
linear isomorphisms and its inverse, composite of a linear transformation.

(d) Representation of Linear transformation by a matrix wrt an ordered bases. Relation betweem matrices of Linear transformation wrt different ordered bases. Matrix of sum, scalar multiple, composite and inverse of Linear transformations.

Equival. lence of rank of a matrix and rank of a linear transformation.
(e)The solution of non homogeneous system of linear equations represented by AX = B. Existance of a solution when rank(A) = rank(AB). The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

Unit 2. Determinants. (15)

(a) Defn of determinant as an n linear skew symmetric function and more egs, determinant of a matrix as determinant of its column vectors or row vectors.

(b) Computation of determinant of n×n matrices, properties such as $\det A_t = \det A$, detAB = detA detB Laplace expansion of a determinant , vandermonde determinant, determinant of
, upper and lower triangular matrices.

(c) Linear dependence and independence of vectors in R_n using determinants, Existence and uniqueness of solution of system AX = B if detA is not 0.

Basic results as A.adj(A) = detA. I.

Crammers rule. Determinant as area and volume.

Unit 3. Eigen values and eigen vectors.(15)

(a) Eigen values and eigen vectors of linear map , Eigen values and eigen vectors of n×n real matrices. Eigen spaces, linear independence of eigen vectors corresponding to distinct eigen values.

(b) The characteristic polynomial of an n×n matrix, characteristic roots.
Similar matrices, characteristic polynomial of similar matrices. characteristic polynomial of linear map.

References:1. Introduction to linear algebra by Serge Lang, Springer verlag. 2. Linear algebra a geometric approach by Kumaresan, Prentice-hall of India private limited, New Delhi. 3. Linear algebra by Hoffman and Kunze, Tata McGraw-Hill, New Delhi.

Computational Mathematics course 4.03

Learning objectives- Introduction to Financial Mathematics. Applications of integrals.

Unit 1.Applications of integrals
(15 Lectures)

(a) Convergence of Integrals. Applications (i) Improper integrals of two types.

(ii) Convergence of improper integrals. Tests of convergence and divergence. Direct form and limit form of the comparison test. Evaluation of convergent improper integrals.

(iii) Applications

1. Finding area of an unbounded region
2. Volumes of solids of revolution of infinite area about x-axis or y- axis.

(iv) Gamma function. Statement of Stirling formula.

Unit-2 Interest rates and options (15 lectures)

(a)Interest Rates and Present Value Analysis:

(i) Present Value Analysis.

(ii) Rate of return.
(iii) Continuously varying Interest Rates.

(b) Options:
(i) Call and Put Options.
(ii) European Options.
(iii) American Options.
(iv) Asian Options.
(v) Options Pricing.

(c) Arbitrage Theory.
(i) Pricing contracts using Arbitrage.
(ii) Multi-period Binomial Model.
(iii) Arbitrage theorem.
(iv) Limitations of Arbitrage Pricing.

Reference of Unit 1: Chapter 12 of Marek Capinski and Tomasz Zastawniak, Probability through Problems , Springer, Indian Reprint 2008 and Chapter 3 - 6 of Sheldon Ross, An elementary introduction to Mathematical Finance, Cambridge University Press second edition 2005

Calculus and Analytic Geometry – Thomos and Finny nineth edition. Addison and Wesley . 1998. chapter 5.1 to 5.6

John C Hull, Options, Futures and other derivatives, Pearson, sixth edition.

Unit 3. Financial Mathematics (Part II)(15 Lectures) **(a) Pricing and Return: Brief Treatment:**

(i) Black Scholes Formula.
(ii) Rates of Return: Single period and Geometric Brownian Motion.
(iii) Pricing American Put Option.
(iv) Portfolio Selection Problem.
(v) Capital Asset Pricing Models.
(vi)Risk Neutral Priced Call Options.

Reference of Unit 2: Chapter 7, Chapter 8 - Section 8.3, Chapter 9 - Sections 9.1, 9.3 - 9.8 of Sheldon Ross, An elementary introduction to Mathematical Finance, Cambridge University Press second edition 2005.

References:
(1) Marek Capinski and Tomasz Zastawniak, Probability through Problems, Springer, Indian Reprint 2008.
(2) Sheldon Ross, An elementary introduction to Mathematical Finance, Cambridge University Press second edition 2005.

(3) Sheldon Ross, A first Course in Probability, Pearson Education, Low Priced edition, 2002.
(4) John C Hull, Options, Futures and other derivatives, Pearson, sixth edition.
(5) David Luenberger, Investment science, Oxford University press.
(6) Paul Wilmott, Paul Wilmott introduces Quantitative Finance, Wiley.

Practicals (S.MAT4.PR)
(1) Convergence of an improper integral
(2) Area and volume of a solid of revolution.
(3) Present value analysis, forward price using arbitrage.
(4) Options: Payoff, Put Call parity
(5) Multi-period Binomial model
(6) Black Scholes option pricing formula
(7) Portfolio selection problem, CAPM (8) Mean variance analysis of a portfolio.

Course: S.Mat.6.01

Title: Real Analysis and Multivariable Calculus

Learning objectives: To understand Differentiability of vector fields, Parametric representation of a surface and Stokes' theorem.

Number of lectures: 45

Unit 1. Differential Calculus

(a) Limits and continuity of vector fields.
Basic results on limits and continuity of sum, difference, scalar multiples of vector fields.
Continuity and components of vector fields.
(b) Differentiability of scalar functions.
(i) Derivative of a scalar field with respect to a non-zero vector.
(ii) Direction derivatives and partial derivatives of scalar fields.
(iii) Mean value theorem for derivatives of scalar fields. (iv) Differentiability of a scalar field at a point (in terms of linear transformation).
Total derivative ,differentiability at a point implies continuity, and existence of direction derivative at the point. The existence of continuous partial derivatives in a neighbourhood of a point implies differentiability at the point.
(v) Chain rule for scalar fields.
(vi) Higher order partial derivatives, mixed partial derivatives.
Sufficient condition for equality of mixed partial derivative.
Second order Taylor formula for scalar fields.

Unit 2. Differentiability of vector fields and its applications.

(i) Gradient of a scalar field. Geometric properties of gradient, level sets and tangent planes.
(ii) Differentiability of vector fields.
(iii) Definition of differentiability of a vector field at a point.
Differentiability of a vector field at a point implies continuity.
(iv) The chain rule for derivative of vector fields.

Unit 3. Parameterization of a surface.

(a) (i) Parametric representation of a surface.
(ii) The fundamental vector product, definition and it being normal to the surface.
(iii) Area of a parametrized surface.

Unit 4. Surface integral.

(a) (i) Surface integrals of scalar and vector fields (definition).
(ii) Independence of value of surface integral under change of parametric representation of the surface.
(iii) Stokes' theorem, (assuming general form of Green's theorem)
Divergence theorem for a solid in 3-space bounded by an orientable closed surface for continuously differentiable vector fields.

List Of Recommended Reference Books

(1) Calculus. Vol. 2, T. Apostol, John Wiley.
(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.
(3) Robert G. Bartle and Donald R. Sherbert. Introduction to Real Analysis, Second edition, John Wiley & Sons, INC.
(4) Richard G. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi.
(5) Tom M. Apostol, Calculus Volume II, Second edition, John Wiley & Sons, New York.

Practicals:

1. Limits and continuity of vector fields, Partial derivative, Directional derivatives.
2. Differentiability of scalar fields.
3. Differentiability of vector fields.
4. Parametrisation of surfaces, area of parametrised surfaces.
5. Surface integrals.
6. Stokes' Theorem and Gauss' Divergence Theorem.
7. Miscellaneous Theoretical questions based on Units 1 and 2.
8. Miscellaneous Theoretical questions based on Units 3 and 4.

COURSE S.Mat.6.02

Title: ALGEBRA

Learning objectives:

Number of lectures: 45

Unit 1. Quotient Spaces (12)

Review of vector spaces over R:

(a) Quotient spaces:

(i) For a real vector space V and a subspace W, the cosets v + W and the quotient space V=W. First Isomorphism theorem of real vector spaces (Fundamental theorem of homomorphism of vector spaces.) (ii) Dimension and basis of the quotient space V=W, when V is finite dimensional.

(b) (i) Orthogonal transformations and isometries of a real finite dimensional inner product space. Translations and reflections with respect to a hyperplane. Orthogonal matrices over R.

(ii) Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space. Characterization of isometries as composites of orthogonal transformations and isometries.

(iii) Orthogonal transformation of R^2. Any orthogonal transformation in R^2 is a reflection or a rotation.

(c) Characteristic polynomial of a square real matrix and a linear transformation of a finite dimensional real vector space to itself. Cayley Hamilton Theorem (Proof assuming the result A adj(A) = In for an square matrix over the polynomial ring R[t].)

Unit 2. Diagonalizability. (10)

(i) Diagonalizability of an real matrix and a linear transformation of a finite dimensional real vector space to itself.

Definition: Geometric multiplicity and Algebraic multiplicity of eigenvalues of an real matrix and of a linear transformation. (ii) matrix A is diagonalisable if and only if R^n has a basis of eigen vectors of A if and only if the algebraic and geometric multiplicities of eigenvalues of A coincide.

(e) Triangularization.

(i) Triangularization of a real matrix having n real characteristic roots.

(f) Orthogonal diagonalization

(i) Orthogonal diagonalization of real symmetric matrices.

(ii) Application to real quadratic forms. Positive definite, semidefinite matrices. Classification in terms of principal minors. Classification of conics in R^2 and quadric surfaces in R^3.

Unit 3. Intrduction to Rings. (14)

(a) (i) Definition of a ring. (The definition should include the existence of a unity element.) (ii) Properties and examples of rings, including Z, Q, R, C, Mn(R), Q[X], R[X], C[X], Z[i], Z[n]. (iii) Commutative ring. (iv) Units in a ring. The multiplicative group of units of a ring. (v) Characteristic of a ring. (vi) Ring homomorphisms. First Isomorphism theorem of rings. Second Isomorphism theorem of rings.

(vii) Ideals in a ring, sum and product of ideals.

(viii) Quotient rings. (b) Integral domains and fields. Definition and examples. (i) A finite integral domain is a field. (ii) Characteristic of an integral domain, and of a finite field. (c) (i) Construction of quotient field of an integral domain (Emphasis on Z, Q). (ii) A field contains a subfield isomorphic to Zp or Q. (d) Prime ideals and maximal ideals. Definition. Examples in Z. Characterization in terms of quotient rings.

Unit 4. Polynomial rings. (9)

Irreducible polynomials over an integral domain. Unique Factorization Theorem for polynomials over a field. (f) (i) Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique Factorization Domain (UFD). Examples of ED: Z, F[X], where F is a field, and Z[i]. (ii) An ED is a PID, a PID is a UFD. (iii) Prime (irreducible) elements in R[X], Q[X], Zp[X]. Prime and maximal ideals in R[X], Q[X]. (iv) Z[X] is a not a UFD (Statement only).

Recommended Books

1. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
2. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. Tom M. Apostol, Calculus Volume 2, Second edition, John Wiley, New York, 1969.
5. W.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi. Additional Reference Books 7) M. Artin. Algebra.

8) N.S. Gopalkrishnan. University Algebra.
9) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Suggested Practicals :

1. Rings, Integral domains and fields.
2. Ideals, prime ideals and maximal ideals.
3. Euclidean Domain, Principal Ideal Domain and Unique Factorization Domain.
4. Quotient spaces.
5. Orthogonal transformations,Isometries.
6. Diagonalization and Orthogonal diagonalization.
7. Miscellaneous Theoretical questions based on Unit 1,2.
8. Miscellaneous Theoretical questions based on Unit 3,4.

Course S.Mat.6.03

Title: Analysis

Learning objectives: Introduction to connectedness and compactness and Fourier Series.

Number of lectures: 45

Unit 1. Compactness (12 lectures)

(a) Definition of a compact set in a metric space (as a set for which every open cover has a finite subcover). Examples, properties such as (i) continuous image of a compact set is compact.

(ii) compact subsets are closed.

(iii) a continuous function on a compact set is uniformly continuous.

(b) Characterization of compact sets in Rn: The equivalent statements for a subset of Rn to be compact:

(i) Heine-Borel property.

(ii) Closed and boundedness property.

(iii) Bolzano-Weierstrass property.

(iv) Sequentially compactness property.

Unit 2. connectedness.(10 lectures)

(c) (i) Connected metric spaces. Definition and examples.

(ii) Different characterizations of a connected space

(iii) Connected subsets of a metric space, connected subsets of R.

(iv) A continous image of a connected set is connected.

(d) (i) Path connectedness in Rn, definitions and examples.

(ii) A path connected subset of Rn is connected.

(iii) An example of a connected subset of Rn which is not path connected.

Unit 3. The function spaces (10 lectures)

(i) The function space C(X;R) of real valued continuous functions on a metric space X. The space C[a; b] with sup norm, Weierstrass approximation Theorem.(Statement only)

(ii) Fourier series of functions on C[-π , π], Bessel's inequality.

Unit 4.Sum of Fourier Series.(13 lectures.)
Dirichlet kernel, Fejer kernel, Cesaro summability of Fourier series of functions on C[-π , π], Pareseval's identity, convergence of the Fourier series in L_2 norm.

List Of Recommended Reference Books

1. S. Kumaresan. Topology of Metric spaces.
2. R.G. Goldberg Methods of Real Analysis, Oxford and IBH Publishing House, New Delhi.
3. W. Rudin. Principles of Mathematical Analysis. McGraw Hill, Auckland, 1976.
4. P.K. Jain, K. Ahmed. Metric spaces. Narosa, New Delhi, 1996.
5. G.F. Simmons. Introduction to Topology and Modern Analysis. McGraw Hill, New York,
6. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
7. E.T. Copson. Metric spaces. Universal Book Stall, New Delhi, 1996.
8. Sutherland. Topology.
9. D. Somasundaram, B. Choudhary. A _rst course in Mathematical Analysis. Narosa, New Delhi.
10. R. Bhatia. Fourier series. Texts and readings in Mathematics (TRIM series), HBA,

Suggested Practicals

1. Compactness in Rn (emphasis on R1, R2). Properties.
2. Connectedness.
3. Path connectedness.
4. Continuous image of compact and connected sets
5 Fourier series;
6 Parseval's identity.
7. Miscellaneous Theoretical Questions based on Unit 1and 2.
8. Miscellaneous Theoretical Questions based on Unit 3 and 4.

Course: S.MAT.6.04 **Title:** Complex Variables

Learning Objectives:-To learn about
i) Analytic functions & integration of such functions
ii) Conformal mapping , cross ratio, Bilinear transformation
iii) Laurent Series, Singularities and its types , Residues , Cauchy`s Residue theorem, Rouche`s theorem

Number of lectures: 45

Unit 1. Complex Numbers (10 Lectures)
Review of complex numbers ,the complex plane,Cartesian-polar-exponential form of a complex number ,Inequalities wrt absolute values, De Moivre`s theorem and its applications, Circular and Hyperbolic functions, Inverse circular and Hyperbolic functions, Separation of real and imaginary parts.

Unit 2. Functions of a Complex Variable(10 Lectures)

Limit ,Continuity ,dirivatives ,analytic functions ,Cauchy-Riemann equations (in cartesian and polar form), harmonic functions, orthogonal curves. To find analytic function when its real/imaginary part or corresponding harmonic function is given. Conformal mapping , cross ratio, Bilinear transformation, fixed(invariant) points.

Unit 3. Complex Integration (10 Lectures)
Rectifiable curves ,integration along piecewise smooth paths , contours.
Cauchy`s theorem & its consequences , Cauchy`s integral formula for derivatives of analytic functions.

Unit 4. Laurent series,Types of singularities & Residues(15 Lectures)
Development of analytic functions as power series–Taylor & Laurent Series.
Entire functions ,Singularities and its types ,Residues , Cauchy`s Residue theorem and its applications-evaluation of standard integrals by Residue calculus method , the Argument principle, Rouche`s theorem & its applications- The Fundamental theorem of Algebra.

Reference Books:-

1) Theory & problems of Complex Variables by Murray R.Spiegel , Schaum`s Outline series McGraw-Hill Book Company,Singapore.
2) Functions of one complex variable by John B.Conway, Narosa Publishing House ,New Delhi.
3) Complex variables and applications by R.V.Churchill
4) Foundations of Complex Analysis by S.Ponnusamy, Narosa Publishing House ,New Delhi.
5) john mathews, russel howell from Narosa

Practicals

1) Analytic functions, Cauchy-Riemann equations, Harmonic functions.
2) Conformal mappings, Bilinear transformations.
3) Integration along piecewise smooth paths, Cauchy`s theorem, Cauchy`s integral formula.
4) Taylor`s & Laurent Series
5) Singularities and its types ,Residues , Cauchy`s Residue theorem and its applications.
6) Rouche`s theorem & its applications
7) Miscellaneous Theoretical questions based on Unit 1 and 2. 8) Miscellaneous Theoretical questions based on Unit 3 and 4.

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Course:S.Mat.6.AC

Title: Computer programming and system analysis

Learning Objectives:-To learn about OOP through java programming, applets

Number of lectures: 50

Unit 1. Java Programming and applets **(16 Lectures)**

Introduction to Classes and Methods(continued)

Defining classes, creating- instance and class variables, creating objects of a class, accessing instance variables of a class, Creating methods, naming methods, accessing methods of class, constructor methods, overloading methods.

Arrays: Arrays (one and two dimensional) declaring arrays, creating array objects, accessing array elements.

Inheritance, interfaces and Packages: Super and sub classes, keywords- "extends", "super‟, „final‟, finalizer methods and overriden methods, abstract classes, concept of interfaces and packages.

Java Applets Basics: Difference of applets and application, creating applets, life cycle of applet, passing parameters to applets.

Graphics, Fonts and Color

The graphics classes, painting the applet, font class, draw graphical figures (oval, rectangle, square, circle, lines,polygones) and text using different fonts.

Unit 2. Networking (09 Lectures)

Introduction

What is networking, need for networking, networking components- nodes, links (point to point and broadcast), networking topologies – bus, star, mesh, network services (connection oriented and connectionless).

Network Design

What is network design, requirement and tasks of a network, LAN MAN, WAN, VAN.

Network Architectures Layering principle, OSI Reference Model, TCP/ IP Reference Model. Comparison of OSI and TCP/P Reference Models.

Network Switching and Multiplexing

Bridges, interconnecting LANs with bridges spanning tree algorithm. What is multiplexing. Static multiplexing (FDM, TDM, WDM), dynamic multiplexing. What is switching, circuit switching, packet switching.

outing and Addressing Router, router table, routing (direct and indirect), routing characteristics, shortest path routing Dijkstra`s algorithm. TCP/IP internetworking, IP addresses (class, classless), and sub netting and subnet mask, Domain names

Unit 3. C Programming. (16 lectures)

Loops and Controls

Control statements for decision making: branching (if statement, if-else statement, else-if statement, switch statement), looping (while loop, do while loop and for loop), breaking out of loops (break and continue statements).

Storage Classes

Automatic variables, external variables, register variables, static variables - scope and functions.

Functions and Arguments

Global and local variables, function definition, return statement, calling a function (by value, by reference), recursion, recursive functions.

Strings and Arrays

Arrays (one and two dimensional), declaring array variables, initialization of arrays, accessing array elements, string functions (strcpy, strcat, strchr, strcmp, strlen, strstr, atoi, atof).Pointers Fundamentals, pointer declarations, operators on pointers, passing pointers to functions, pointers and one dimensional array, pointers and two dimensional array. Structures.Basics of structures, structures and functions.

Unit 4. Introduction to DBMS and RDBMS (9 Lectures)

Introduction to Database Concepts

Database systems vs file systems, view of data, data models, data abstraction, data independence,

three level architecture, database design, database languages - data definition language(DDL), data manipulation language(DML). E - R Model

Basic concepts, keys, E-R diagram, design of E-R diagram schema (simple example).

Relational structure

Tables (relations), rows (tuples), domains, attributes, candidate keys, primary key, entity integrity constraints, referential integrity constraints, query languages, normal forms 1,2,and 3 (statements only), translation of ER schemas to relational (database) schemas (logical design), physical design.

Recommended Books:-

(1)The complete reference java2: Patrick maughton, Hebert schind (TMH).
(Chapters 1 – 6, 8-9, 12, 21)

(2)Computer Networks – Andrew S. Tanenbaum (PHI) (Chapter 1: 1.1-1.4, chapter 2:2.5.4.2.5.5 Chapter 5:P 5.2.1-5.2.4,5.5.1-5.5.2,5.6.1-5.6.2, Chapter 7:7.1.1. 7.1.3).

(3)Programming in Ansi C - Ram Kumar and Rakesh Agarwal (Tata McGraw Hill) (Chapters 2 - 8).

(4) Database System Concepts - Silberschatz, Korth, Sudarshan (McGraw-Hill Int. Edition) 4th Edition (Chapter 1: 1.1 - 1.5, Chapter 2: 2.1 - 2.5, 2.8 - 2.9, Chapter 3: 3.1, Chapter 7: 7.1,7.2, 7.7)

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Practicals:- Java programs that illustrate

1) the concept of java class
   (i) with instance variable and methods
   (ii) with instance variables and without methods
   (iii) without instance variable and with methods

   Create an object of this class that will invoke the instance variables and methods accordingly.
2) the concept of (one dimensional) arrays
3) the concept of (two dimensional) arrays
4) the concept of java class that includes inheritance
5) the concept of java class that includes overridden methods
6) the concept of java class that includes interfaces and packages
7) applets
8)Java programs on numerical methods.

C programs for:
1. Creating and printing frequency distribution.
2. (a) Sum of two matrices of order m × n and transpose of a matrix of order m × n, where m, n = 3.

(b) Multiplication of two matrices of order m, where m = 3, finding square and cube of a square matrix using function.

3. Simple applications of recursive functions (like Factorial of a positive integer, Generating

Fibonacci Sequence, Ackerman Function, univariate equation)

4. Sorting of Numbers (using bubble sort, selection sort), and strings.
5. Using arrays to represent a large integer (that cannot be stored in a single integer variable).
6. Counting number of specified characters (one or more) in a given character string.
7. Writing a function to illustrate pointer arithmetic.
8. Using structures to find and print the average marks of five subject along with the name of a student.
9. Program to find g.c.d. using Euclidean algorithm.
10. Numerical methods with C programs.
11. Program to decide whether given number is prime or not.

12. Finding roots of quadratic equation using C program.
13. Programs to find trace, determinant of a matrix.
14. Program to check given matrix is symmetric or not.

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