St. Xavier's College (Autonomous), Mumbai

Syllabus of the courses offered by the Department of Mathematics

(2018-19)

St. Xavier's College – Autonomous Mumbai

Syllabus
For ODD Semester Courses in
MATHEMATICS
(2018-2019)

Contents:

Theory Syllabus for Courses:

F.Y.B.Sc. – Mathematics **Course Code: S.MAT.1.01**

Title: CALCULUS – I

Learning Objectives: To learn about (i) lub axiom of R and its consequences.
(ii) Convergence of sequences in R.
(iii) Limit and continuity of real valued functions of one variable.

Number of lectures : 45

Unit-I: Real Number System and Sequence of Real Numbers (15 Lectures)

Real Numbers: Real number system and order properties of R, Absolute value properties AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhood, Hausdorff property, Bounded sets, Continuum property (l.u.b.axiom–statement, g.l.b.) and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and its applications, Density theorem.

Sequences: Definition of a sequence and examples, Convergence of a sequence, every convergent sequence is bounded, Limit of a sequence, uniqueness of limit if it exists, Divergent sequences, Convergence of standard sequences like $\{n^{1/n}\}$, $\{a^n\}$ Sequential version of Bolzano-Weierstrass theorem.

Unit II: Sequences (contd.) (15 Lectures)

Algebra of convergent sequences, Sandwich theorem for sequences, Monotone sequences, Monotone Convergence theorem and its consequences such as convergence of $\left(1+\frac{1}{n}\right)^n$.

Subsequences: Definition, Subsequence of a convergent sequence is convergent and converges to the same limit.

Cauchy sequence: Definition, every convergent sequence is a Cauchy sequence and converse.

Unit III: Limits and Continuity of real valued functions of one variable (15 Lectures)

Limit of Functions: Graphs of some standard functions such as |x|, e^x, log x, $\frac{1}{x}$, ax^2+bx+c, x^3, x⌋ (Flooring function), ┌x┐ (Ceiling function), sin x, cos x, tan x, x sin(1/x), x^2 sin(1/x) over suitable intervals, Graph of a bijective function and its inverse, Limit of a function, evaluation of limit of simple functions using $\in -\delta$ definition, uniqueness of limit if it exists, Algebra of limits (with proof), Limit of a composite functions, Sandwich theorem (only statement), Left hand and right hand limits, non–existence of limits, Limit as $x \to \pm\infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, $\in -\delta$ definition of continuity, Sequential continuity, Algebra of continuous functions, Continuity of composite functions. Discontinuous functions, examples of removable and essential discontinuity.

Recommended Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. Robert G. Bartle and Donald R. Sherbet : Introduction to Real Analysis, Springer Verlag.
4. Howard Anton, Calculus - A new Horizon, Sixth Edition, John Wiley and Sons Inc,1999.

Additional Reference Books

1. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd.
2. Courant and John, A Introduction to Calculus and Analysis, Springer.
3. Ajit and Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company,1994.
5. Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2000.

Assignments (Tutorials)

1. Application based examples of Archimedean property, intervals, neighbourhood.
2. Consequences of continuum property, infimum and supremum of sets.
3. Calculating limits of sequence.
4. Cauchy sequence, monotone sequence.
5. Limit of a function and Sandwich theorem.
6. Continuous and discontinuous functions.

F.Y.B.Sc. – Mathematics **Course Code: S.MAT.1.02**

Title: ALGEBRA - I

Learning Objectives: To learn about (i) divisibility of integers.
(ii) properties of equivalence relations and partitions.
(iii) roots of polynomials.

Number of lectures : 45

Prerequisites:

Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations and combinations.

Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit I: Integers and divisibility **(15 Lectures)**

Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal Triangle. Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a and b and that the g.c.d. can be expressed as ma + nb where m, n are in Z, Euclidean algorithm, Primes, Euclid's lemma, Fundamental theorem of arithmetic, the set of primes is infinite. Congruences, definition and elementary properties, Euler's ϕ function, Statements of Euler's theorem, Fermat's theorem and Wilson theorem, Applications.

Unit II: Functions and Equivalence relations **(15 Lectures)** Definition of a function, domain, codomain and range of a function, composite functions, examples, Direct image f [A] and inverse image $f^{-1}[A]$ of a function. Injective, surjective, bijective functions, Composite of injective, surjective, bijective functions, Invertible functions, Bijective functions are invertible and conversely, Examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples. Equivalence relations, Equivalence classes, properties such as two equivalences classes are either identical or disjoint. Definition of partition, every partition gives an equivalence relation and vice versa, Congruence an equivalence relation on Z, Residue classes, Partition of Z, Addition modulo n, Multiplication modulo n, examples, conjugate classes.

Unit III: Polynomials **(15 Lectures)**

Definition of polynomial, polynomials over F where F = Q , R ,C. Algebra of polynomials, degree of polynomial, basic properties, Division algorithm in F[X](without proof) and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem, A polynomial of degree n over F has at most n roots. Complex roots of a polynomial in R[X] occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree n in R[X] has exactly n complex roots counted with multiplicity. A non-constant polynomial in R[X] can be expressed as a product of linear and quadratic factors in C[X]. Necessary condition for a rational number to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is an irrational number where p is a prime number, n^{th} roots of unity, sum of n^{th} roots of unity.

Recommended Books

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education
(India) Private Ltd.
2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

Additional Reference Books

1. I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.
2. G. Birkoff and S. Maclane, A Survey of Modern Algebra, Third Edition, Mac Millan, New York, 1965.
3. N. S. Gopalkrishnan, University Algebra, Ne Age International Ltd, Reprint, 2013.
4. I .N. Herstein, Topics in Algebra, John Wiley, 2006.
5. P. B. Bhattacharya S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International, 1994.
6. Kenneth Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition, Mathematics Series.

Assignments (Tutorials)

1. Mathematical induction (The problems done in F.Y.J.C. may be avoided).

2. Division Algorithm and Euclidean algorithm, in Z, Primes and the Fundamental Theorem of Arithmetic.
3. Functions (direct image and inverse image). Injective, surjective, bijective functions, finding inverses of bijective functions.
4. Congruences and Euler's function, Fermat's little theorem, Euler's theorem and Wilson's theorem.
5. Equivalence relation.
6. Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

CIA I – 20 marks, 45 mins. (Objectives/Short questions, not more than 5 marks each)

CIA II – 20 marks, 45 mins. (Objectives/Short questions, not more than 5 marks each)

End Semester exam – 60 marks, 2 hours.

There will be three questions, one per unit. The Choice is internal- i.e. within a unit and could be between 50% to 100%

S.Y.B.Sc. – Mathematics **Course Code: S.MAT.3.01**

Title: CALCULUS – III

Learning Objectives: To learn about (i) Riemann Integration **(ii) Different coordinate systems, Sketching in Improper integrals, β and Γ functionsR^2 and R^3, iii) double integrals and its applications**

Number of lectures : 45

Unit I: Riemann Integration (15 Lectures)

Approximation of area,Upper / Lower Riemann sums and properties, Upper / Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion for Riemann integrability, For a < c < b, f ∈ R[a, b] if and only if f ∈ R[a, c] and f ∈ R[c, b] with $\int_a^b f = \int_a^c f + \int_c^b f$. Properties: f, g ∈ R[a, b] then λ f ∈ R[a, b] and f+g ∈ R[a, b] with $\int_a^b \lambda f = \lambda \int_a^b f$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$ and; f ∈ R[a, b] $|f| \in R[a, b]$; $\left|\int_a^b f\right| \leq \int_a^b |f|$;

$f \geq 0 \int_a f \geq 0$; f ∈ C[a, b] f ∈ R[a, b]; if f is bounded with finite number of discontinuities then f ∈ R[a, b] ; generalize this if f is monotone then f ∈ R[a, b].

Unit II: Indefinite and improper integrals (15 Lectures)

Continuity of $F(x) = \int_a^x f(t)dt$ where f ∈ R[a, b], Fundamental theorem of calculus,Mean value theorem, Integration by parts, Leibnitz rule, Improper integrals – type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abel's and Dirichlet's tests (without proof); β and Γ functions with their properties, relationship between β and Γ functions.

Unit III: Applications (15 Lectures)

Topics from analytic geometry – sketching of regions in R^2 and R^3 , graph of a function, level sets, Cylinders and Quadric surfaces, Cartesian coordinates, Polar coordinates, Spherical coordinates, Cylindrical coordinates and conversions from one coordinate system to another.

(a) Double integrals: Definition of double integrals over rectangles, properties, double integrals

over a bounded region.

(b) Fubini`s theorem (without proof) – iterated integrals, double integrals as volume.

(c) Application of double integrals: average value, area, moment, center of mass.

(d) Double integral in polar form.

(Reference for Unit III: Sections 5.1, 5.2, 5.3 and 5.5 from Marsden-Tromba-Weinstein).

References:

1. Goldberg, Richard R.: Methods of real analysis. (2nd ed.) New York. John Wiley & Sons, Inc., 1976. 0-471-31065-4--(515.8GOL)

2. Goldberg, Richard R.: Methods of real analysis. (1st ed. Indian Reprint) New Delhi. Oxford & IBH Publishing Co., 1964(1975).--(515.8GOL)

3. Kumar, Ajit & Kumaresan, S.: A basic course in real analysis. (Indian reprint)

4. Boca Raton. CRC Press, 2015. 978-1-4822-1637-0--(515.8Kum/Kum)

5. Apostol, Tom M.: Calculus, Vol.-II. [Multi-variable calculus and linear algebra, with applications to differential equations and probability] (2nd ed.) New York. John Wiley & Sons, Inc., 1969. 0-471-00008-6--(515.14APO)

6. Stewart, James: Multivariable calculus:concepts & contexts. Pacific Grove.Brooks/Cole Publishing Company, 1998. 0-534-35509-9--(515STE)

7. Marsden, Jerrold E.;Tromba, Anthony J. & Weinstein, Alan: Basic multivariablecalculus.

(Indian reprint) New Delhi. Springer (India) Private Limited, 1993(2004). 81-8128-186-1--(515.84MAR)

8. Robert G. & Sherbert, Donald R.: Introduction to real analysis. (3rd ed.) New Delhi. Wiley India (P) Ltd, 2005(2007). 81-265-1109-5--(515.8Bar/She)

Suggested Tutorials:

1. Calculation of upper sum, lower sum and Riemann integral.
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests.
5. Sketching of regions in R^2 and R^3 , graph of a function, level sets, Cylinders and Quadric surfaces, conversions from one coordinate system to another.
6. Double integrals, iterated integrals, applications to compute average value, area, moment, center of mass.

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S.Y.B.Sc. – Mathematics **Course Code: S.MAT.3.02**

Title: ALGEBRA - III

Learning Objectives: (i) To understand linear maps and isomorphism between vector spaces

(ii) To study determinant function via permutations and its Laplace expansion,

Inverse of a matrix by adjoint method, Cramer`s rule,

(iii) To study Gram–Schmidt orthogonalization process in an inner product space.

Number of lectures : 45

Unit I: Linear Transformations and Matrices (15 Lectures)

1. Review of linear transformations and matrix associated with a linear transformation: Kernel and image of a linear transformation, Rank– Nullity theorem (with proof), Linear isomorphisms and

its inverse. Any n–dimensional real vector space is isomorphic to RMatrix of sum, scalar multiple, composite and inverse of Linear transformations$_n$, Sum and scalar multiple of a linear.

transformation, spaceL(U,V) of Linear transformation from U to V where U and V are finite dimensional vector spaces over R , the dual space V*, linear functional, linear operator.

2. Elementary row operations, elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices.

3. Row space, column space of an m× n matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, Invariance of rank upon elementary row or column operations.

4. Equivalence of rank of an m× n matrix and rank of the linear transformation L_A: $R^n \rightarrow R^m$ $L_A(X) = AX$ where A is an m× n matrix. The dimension of solution space of the system of linear equation AX = O equals n–rank(A).

5. The solutions of non – homogeneous systems of linear equations represented by AX = B, Existence of a solution when rank(A)= rank(A, B). The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

Unit II: Determinants (15 Lectures)

1. Definition of determinant as an n–linear skew–symmetric function from $R^n \times R^n \times \ldots \times R^n \rightarrow R$ such that determinant of $(E_1, E_2, \ldots, E_n)$ is 1 where E_j denotes the j^{th} column of the identity matrix I_n. Determinant of a matrix as determinant of its column vectors (or row vectors).

2. of 2 Existence and uniqueness of determinant function via permutations, Computation of determinant $\times$ 2, 3$\times$ 3 matrices, diagonal matrices, Basic results on determinants such as det(A)=det(A_t),

det(AB) = det(A) det(B). Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices, finding determinants by row reduction method.

3. Linear dependence and independence of vectors in R^n using determinants. The existence and uniqueness of the system AX = B where A is an n $\times$ n matrix with det(A) $\neq$ 0. Cofactors and minors, Adjoint of an n$\times$ n matrix. Basic results such as A (adj A) = det(A) I_n. An n$\times$ n real matrix is invertible if and only if det(A) $\neq$ 0. Inverse (if exists) of a square matrix by adjoint method, Cramer's rule.

4. Determinant as area and volume.

Unit III: Inner Product Spaces (15 Lectures)

1. Dot product in R^n, Definition of general inner product on a vector space over R. Examples of inner product including the inner product $\langle f, g\rangle = \int_{-\pi}^{\pi} f(t)g(t)dt$ on C[− π, π], the space of continuous real valued functions on [− π, π].

2. Norm of a vector in an inner product space. Cauchy–Schwarz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras's theorem and geometric applications in R^2, Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in R^2 and R^3. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases, Gram–Schmidt orthogonalization process, Simple examples in R^2, R^3, R^4.

References:

1. Lang, Serge: Introduction to linear algebra. (2nd ed. 3rd Indian reprint) New Delhi. Springer (India) Private Limited, 2005(2009). 81-8128-260-6--(512.5Lan)

2. Kumaresan, S.: Linear algebra : a geometric approach. New Delhi. Prentice-Hall Of India Private Limited, 2001. 81-203-1628-2--(512.5KUM)

3. Krishnamurthy, V.; Mainra, V.P. & Arora, J.L: Introduction to linear algebra. New Delhi.Affiliated East-West Press Pvt. Ltd., 1976. 81-85095-15-9--(512.5KRI)

4. Lay, David C.: Linear algebra and its applications. (3rd ed.) Noida. Pearson Education Inc., 2016. 978-81-775-8333-5--(512.5Lay)

5. Artin, Michael: Algebra. (2nd ed. Indian Reprint) New Delhi. Prentice-Hall Of India Private Limited, 1994. 81-203-0871-9--(512ART)

6. Hoffman, Kenneth and Kunze, Ray: Linear algebra. (2nd ed.) Noida. Pearson India Education Services Pvt. Ltd, 2015. 978-93-325-5007-0--(512.5Hof)

7. Strang, Gilbert: Linear algebra and its applications. (3rd ed.) Fort Worth. Harcourt Brace Jovanovich College Publishers, 1988. 0-15-551005-3--(512.5STR)

8. Smith, Larry: Linear algebra. (3rd ed.) New York. Springer-Verlag, 1978.0-387-98455-0-- (512.5SMI)

9. Rao, Ramachandra A. and Bhimasankaran, P.: Linear Algebra. New Delhi. Tata Mcgraw- Hill Publishing Co. Ltd., 1992. 0-07-460476-7--(512.5RAO)

10. Banchoff, Thomas & Wermer, John: Linear algebra through geometry. (2nd ed.) New York. Springer-Verlag, 1984. 0-387-97586-1--(512.5BAN/WER)

11. Axler, Sheldon: Linear algebra done right. (2nd ed.) New Delhi. Springer (India) Private Limited, 2010. 81-8489-532-2--(512.5Axl)

12. Janich, Klaus: Linear algebra.New Delhi. Springer (India) Private Limited, 1994. 978-81- 8128-187-6--(512.5Jan)

13. Bretscher, Otto: Linear algebra with applications.(3rd ed.) New Delhi. Dorling Kindersley (India) Pvt. Ltd, 2008. 81-317-1441-6--(512.5Bre)

14. Williams, Gareth: Linear algebra. New Delhi. Narosa Publishing House, 2009. 81-7319- 981-3--(512.9Wil)

Suggested Tutorials:

1. Rank–Nullity Theorem.
2. System of linear equations.

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3. Calculating determinants of matrices, diagonal and upper / lower triangular matrices using definition, Laplace expansion and row reduction method, Linear dependence / independence of vectors by determinant concept.
4. Finding inverses of square matrices using adjoint method, Examples on Cramer's rule.
5. Examples of Inner product spaces, Orthogonal complements in R^2 , R^3, R^4. and R^3 .
6. Examples on Gram–Schmidt process in R ^{2}

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S.Y.B.Sc. – Mathematics **Course Code: S.MAT.3.03**

Title: FINITE MATHEMATICS

Learning Objectives: To learn about (i) Advanced counting
(ii) Permutation and Recurrence Relation
(iii) Introductory Graph theory.
(iv) Introduction to probability measure

Number of lectures : 45

Unit I: Counting (15 Lectures)

1. Finite and infinite sets, Countable and uncountable sets, examples such as N, Z, $N\times N$, Q, $(0,1)$, R .
2. Addition and multiplication principle, Counting sets of pairs, two ways counting.
3. Stirling numbers of second kind, Simple recursion formulae satisfied by $S(n,k)$ and direct formulae for $S(n,k)$ for $k = 1,2,\cdots,n$.
4. Pigeon hole principle and its strong form, its applications to geometry, monotonic sequences.
5. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities
6. Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
7. Non-negative and positive integral solutions of equation $x_1 + x_2 + \cdots + x_k = n$.
8. Principle of Inclusion and Exclusion, its applications, derangements, explicit formula for d_n , various identities involving d_n , deriving formula for Euler's phi function $\varphi(n)$.

9. Permutation of objects, s_n composition of permutations, results such as every permutation is product of disjoint cycles; every cycle is product of transpositions, even and odd permutations, rank and signature of permutation, cardinality of S_n, A_n.

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10. Recurrence relation, definition of homogeneous, non-homogeneous, linear and non-linear recurrence relation, obtaining recurrence relation in counting problems, solving (homogeneous as well as non-homogeneous) recurrence relation by using iterative method, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Unit II: Graph Theory (15 Lectures)

1. Definition, examples and basic properties of graphs, pseudographs, complete graphs, bipartite graphs.
2. Isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, The adjacency matrix, weighted graph.
3. Travelling salesman's problem, shortest path, Floyd-Warshall algorithm, Dijkstra's algorithm.

Unit III: Probability Theory (15 Lectures)

1. Finitely Additive Probability Measure.
2. Sigma Fields.
3. Discrete Random Variables.
4. Joint Distribution of Random Variables as a Probability Measure.
5. Expectation and Variance of Discrete Random Variables.
6. Conditional Expectation.
7. Chebyshev's Inequality, Weak Law of Large Numbers and Central Limit Theorem.

Recommended books:

1. Biggs, Norman L.: Discrete mathematics. (Revised ed.) Oxford. Oxford University Press, 1993. 0-19-853427-2--(510BIG)

2. Brualdi, Richard A.: Introductory combinatorics. (3rd Ed.) Upper Saddle River. Prentice-Hall, Inc., 1999. 0-13-181488-5--(511.6BRU)

3. Goodaire, Edgar and Michael Parmenter, Michael: Discrete Mathematics with Graph

Theory, Pearson.(3rd ed.)

4. Rosen, Kenneth H.: Discrete mathematics and its applications. (5th ed.) New Delhi. Tata Mc-Graw Hill Publishing Company Limited, 2003. 0-07-242434-6--(511ROS)

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5. Marek Capinski and Tomasz Zastawniak : Probability through Problems, Springer.

<u>Reference Books:</u>

1. Krishnamurthy, V.: Combinatorics: Theory and applications. New Delhi. Affiliated East-West Press Pvt. Ltd., 1985. 81-85336-02-4--(510KRI)

2. Rosen, Kenneth H.: Discrete mathematics and its applications. (5th ed.) New Delhi. Tata Mc-Graw Hill Publishing Company Limited, 2003. 0-07-242434-6--(511ROS)

3. Lipschutz, Seymour & Lipson, Marc Lars: Theory and problems of discrete mathematics.
(2nd ed. Indian reprint) New Delhi. Tata Mcgraw-Hill Publishing Co. Ltd., 1997(1999). 0-07-463710-X--(512LIP/LIP)

4. Allen Tucker, Allen: Applied Combinatorics, (6th ed.), John Wiley and Sons.

5. Grimaldi, Ralph P.: Discrete and combinatorial mathematics : an applied introduction. (3rd ed.) Reading. Addison-Wesley Publishing Company, 1994.
0-201-54983-2--(510GRI)

6. Rosen, Kenneth H.: Discrete mathematics and its applications with combinatorics and graph theory. (7th ed.) New Delhi. McGraw Hill Education (India) Private Ltd., 2011(2015). 978-0-07-068188-0--(511Ros)

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<u>PRACTICALS: Course Code :S.MAT.3.PR FINITE MATHEMATICS</u>

1. Problems based on counting principles, Two way counting.

2. Stirling numbers of second kind, Pigeon hole principle.

3. Multinomial theorem, identities, permutation and combination of multi-set.

4. Inclusion-Exclusion principle, Euler phi function.

5. Derangement and rank signature of permutation, Recurrence relation

6. Drawing a graph, checking if a degree sequence is graphical. Representing a given graph by an adjacency matrix and drawing a graph having given matrix as adjacency matrix.

7. Problems on Discrete random variables, expectation and variance.

8. Problems Based on Chebyshev's Inequality, Weak Law of Large Numbers and Central Limit Theorem.

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EVALUATION:-

CIA- I: 20 marks, 45 mins.
Unit I: Objectives/Short questions, not more than 5 marks each

CIA- II : 20 marks, 45 mins.
Unit II: Short questions/Presentation/Assignment, not more than 5 marks each

End Semester Exam – 60 marks, 2 hours OR Additional Exam – 100 marks, 3 hours

T.Y.B.Sc. - Mathematics Course Code: S.MAT.5.01

Title of Paper: CALCULUS - V

Learning Objectives:
After completion of the course, a student should:
(i) Learn about Multiple, Line and Surface integrals.
(ii) Understand the differences between above mentioned integrals.
(iii) Apply the concepts on different physical problems. For e.g., finding centre of gravity, moment of inertia and flux.

Number of lectures: 45

Unit I: Multiple Integrals - I (12 Lectures)

Definition of double (respectively: triple) integral of a function bounded on a rectangle (respectively: box), Geometric interpretation as area and volume.
Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals.

Basic properties of double and triple integrals proved using the Fubini's theorem such as; Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions, Formulae for the integrals of sums and scalar multiples of integrable functions, Integrability of continuous functions.

Unit II: Multiple Integrals - II (11 Lectures)

Integrability of bounded functions having finite number of points of discontinuity, Domain additivity of the integral.
Integrability and the integral over arbitrary bounded domains.
Change of variables formula (Statement only), Polar, cylindrical and spherical coordinates and integration using these coordinates.
Differentiation under the integral sign.
Applications to finding the center of gravity and moments of inertia.

Unit III: Line Integrals (11 Lectures)

Review of Scalar and Vector Fields on $\mathbb{R}$, Vector Differential Operators.
Gradient Paths (Parametrized Curves) in $\mathbb{R}$, Smooth and piecewise smooth paths. Closed paths, Equivalence and orientation preserving equivalence of the paths.
Definition of the line integral of a vector field over a piecewise smooth path, Basic properties of line integrals including linearity, path-additivity and behavior under a change of parameters, Examples.
Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be conservative.
Flux across a plane curve.
Green's Theorem (proof in the case of rectangular domains).
Applications to evaluation of line integrals.

Unit IV: Surface Integrals (11 Lectures)

Parameterized surfaces. Smoothly equivalent parameterizations, Area of such surfaces. Definition of surface integrals of scalar-valued functions as well as of vector fields defined on a surface.
Independence of value of surface integral under change of parametric representation of the surface.
Curl and divergence of a vector field, Elementary identities involving gradient, curl and divergence.
Stokes' Theorem (proof assuming the general form of Green's Theorem), Examples.
Gauss' Divergence Theorem (proof only in the case of cubical domains), Examples.

References:
1. Tom V. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969
2. James Stewart, Calculus: Early transcendental Functions
3. J. E. Marsden and A.J. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co.,

New York, 1996
4. Lawrence Corwin and Robert Szczarba, Multivariable Calculus

Additional References:

1. T Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi. 1947.
2. R. Courant and F. John, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.
3. Wendall Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.
4. M. H. Protter and C. B. Morrey, Jr., Intermediate Calculus, Second Ed., SpringerVerlag, New York, 1995.
5. G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
6. David Widder Advanced Calculus, Second Ed., Dover Pub., New York. 1989.
7. Sudhir R. Ghorpade and Balmohan Limaye, A course in Multivariable Calculus and Analysis, Springer International Edition.
8. M. J. Strauss, G. L. Bradley and K. J. Smith, Calculus (3rd Edition), Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2007.
9. Michael Spivak, Calculus on Manifolds.
10. Jerrold E. Marsden, Anthony J. Tromba, Alan Weinstein, Basic Multivariable Calculus.

Suggested Practicals (3 practicals per batch per week):

1. Evaluation of double and triple integrals.
2. Change of variables in double and triple integrals and applications.
3. Line integrals of scalar and vector fields.
4. Green's theorem, conservative field and applications.
5. Evaluation of surface integrals.
6. Stokes' and Gauss' Divergence Theorem.

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T.Y.B.Sc. – Mathematics Course Code: S.MAT.5.02

Title: ALGEBRA V

Learning Objectives (i) To learn Diagonalization of matrices.

(ii) To learn Quotient spaces and Orthogonal transformations.

Number of lectures: 45

Unit I. Quotient Spaces and Cayley Hamilton Theorem (11L)

Review of vector spaces over $\mathbb{R}$, sub spaces and linear transformation.Quotient Spaces: For a real vector space and a subspace , the cosets and the quotient space , First Isomorphism theorem of real vector spaces(fundamental theorem of homomorphism of vector spaces), Dimension and basis of the quotient space

,when is finite dimensional. Characteristic polynomial of Real matrix. Cayley Hamilton theorem and its applicatios.

Reference for Unit I:

(i) S. Kumaresan, Linear Algebra: A Geometric Approach.
(ii) Inder K Rana, Introduction to Linear Algebra, Ane Books Pvt. Ltd. Inder K Rana, Introduction to Linear Algebra, Ane Books Pvt. Ltd.
(ii) Ramachandra Rao and P. Bhimasankaram, Tata McGraw Hill Publishing Company.

Unit II. Orthogonal transformations (11 L)

Isometries of a real finite dimensional inner product space, Translations and Reflections with respect to a hyperplane, Orthogonal matrices over $\mathbb{R}$, Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space, Orthogonal transformation of $\mathbb{R}$, Any orthogonal transformation in $\mathbb{R}$ is a reflection or a rotation, Characterization of isometries as composites of orthogonal transformations and translation.

Reference for Unit II:

(i) K Hoffman and Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
(ii) S. Kumaresan, Linear Algebra: A Geometric Approach.

Unit III. Eigenvalues and eigen vectors (11L)

Eigen values and eigen vectors of a linear transformation T: V→V ,where V is a finite dimensional real vector space and examples, Eigen values and Eigen vectors of $\times$ real matrices, The linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation and a Matrix. The characteristic polynomial of a $\times$ real matrix and a linear transformation of a finite dimensional real vector space to itself, characteristic roots, Similar matrices, Relation with change of basis, Invariance of the characteristic polynomial and (hence of the) eigen values of similar matrices, Every square matrix is similar to an upper triangular matrix. Minimal Polynomial of a matrix, Examples like minimal polynomial of scalar matrix, diagonal matrix, similar matrix, Invariant subspaces.

Reference for Unit III:

(i) K Hoffman and Kunze, Linear Algebra, Prentice Hall of India, New Delhi.

Unit IV: Diagonalisation (12L)

Geometric multiplicity and Algebraic multiplicity of eigen values of a nxn real matrix. Matrix is diagonalizable if and only if it has basis containing eigen vectors if and only if sum of dimensions of eigen spaces is n if and only if their geometric and algebraic multiplicities coincide. Examples of non diagonalizable matrices. Diagonalisation of a linear transformation.Orthogonal diagonalisation and Quadratic Forms. Diagonalisation of real Symmetric matrices, Examples, Applications to real Quadratic forms, Rank and Signature of a Real Quadratic form, Classification of conics in $\mathbb{R}$ and quadric surfaces in

$\mathbb{R}$. Positive definite and semi definite matrices, Characterization of positive definite matrices in terms of principal minors.

Reference for Unit IV:

(i) S. Kumaresan, Linear Algebra: A Geometric Approach.
(ii) K Hoffman and Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
Recommended Books:

(i) S. Kumaresan, Linear Algebra: A Geometric Approach.
(ii) K Hoffman and Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
(iii)Inder K Rana, Introduction to Linear Algebra, Ane Books Pvt. Ltd.

Additional Reference Books:
(i) Ramachandra Rao and P. Bhimasankaram, Tata McGraw Hill Publishing Company.

(ii) T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.
(iii) L. Smith, Linear Algebra, Springer.
(iv) M. R. Adhikari and Avishek Adhikari, Introduction to linear Algebra, Asian Books Private Ltd.
(v) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.

Suggested Practicals:

1)Quotient Spaces, Orthogonal Transformations.

2) Cayley Hamilton Theorem and Applications

3) Eigen Values & Eigen Vectors of a linear Transformation/ Square Matrices

4) Similar Matrices, Minimal Polynomial, Invariant Subspaces

5) Diagonalisation of a matrix

6) Orthogonal Diagonalisation and Quadratic Forms.

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Title: Topology of Metric Spaces I Course Code: S.MAT.5.03

Learning Objective: Introduction to Metric Spaces.

Unit I. Metric Spaces (10 L)

Definition, examples of metric spaces, $\mathbb{R}$ with usual distance, the Euclidean space $\mathbb{R}^2$, sup and sum metric, discrete metric, the metric space of complex numbers.The metric induced by a norm, translation invariance of the metric induced by the norm. Metric subspaces, Product of two metric spaces. Open balls and open set in a metric space, examples of open sets in various metric spaces, Hausdorff property, Interior of a set, Properties of open sets, Structure of an open set in $\mathbb{R}$, Equivalent metrics.

Unit II. Closed Sets, Distance of a point from a Set, Distance between Sets (10 L)
Closed ball in a metric space, Closed sets- definition, examples. Limit point of a set, closure point, isolated point. A closed set contains all its limit points, Closure of a set and boundary, Sequences in a metric space, Distance of a point from a set, distance between two sets, diameter of a set, Bounded sets.

Unit III. Sequences (10 L)

Convergent sequence in a metric space, Cauchy sequence in a metric space, subsequences, examples of convergent and Cauchy sequences in metric spaces,

$\mathbb{R}$ with different metrics and other metric spaces. Characterization of limit points and closure points in terms of sequences. Definition and examples of relative openness,

Unit IV. Compact sets (15 L)

Definition of compact metric space using open cover, examples of compact sets in different metric spaces. Properties of compact sets—compact set is closed and bounded, every infinite bounded subset of a compact metric space has a limit point, Heine- Borel theorem. Equivalent statements for compact sets in $\mathbb{R}^n$; Heine- Borel property, Closed and boundedness property, Bolzano- Weierstrass property, Sequential compactness property.

Recommended Books :
1. P.K. Jain and Khalil Ahmed,Metric Spaces
2. S. Kumaresan, Topology of Metric spaces.
3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.

Additional Reference Books.
1. W. Rudin, Principles of Mathematical Analysis.

2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974

3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.

4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.

Suggested Practicals:
1. Metric spaces.
2. Open sets , Closed sets in Metric spaces.
3. Sequences

4. Cauchy sequence, subsequences
5. Compact sets
6. Bolzano- Weierstrass property, Sequential compactness property

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T.Y. B.Sc. Mathematics Course Code: S.Mat.5.04

Title: Numerical Methods-I

Learning Objectives: To learn about (i) Iteration methods based on first/second degree equation.

(ii) Iteration methods for polynomial equations.

(iii) solving a system of linear algebraic equations.

(iv) Eigen Value Problem

Number of lectures: 45

Unit I. Errors Analysis, Transcendental and Polynomial Equations-I (12 L)

Measures of Errors: Relative, absolute and percentage errors. Types of errors: Inherent error, Round-off error and Truncation error. Examples using Taylor's series Significant digits and numerical stability. Concept of simple and multiple roots. Direct and Iterative methods, error tolerance, use of intermediate value theorem and finding initial approximation of a root. Iteration methods based on first degree equation: Regula-Falsi method, Secant method, Newton-Raphson method, General Iteration Method. Condition of convergence and Rate of convergence of all above methods. Methods for multiple roots.

Unit II. Transcendental and Polynomial Equations-II (11 L)

Iteration methods based on second degree equation: Muller method, Chebyshev method, Multipoint iteration method and their rate of convergence.

Iterative methods for polynomial equations: Descart's rule of signs, Sturm sequence, BirgeVieta method, Bairstow method, Graeffe`s root squaring method.

Unit III. System of Linear Algebraic Equations (11 L)

Matrix representation of linear system of equations.

Direct methods: Gauss elimination method. Pivot element, Partial and complete pivoting, Forward and backward substitution method, Triangularization methods-Doolittle`s and Crout`s method, Cholesky's method, LU decomposition, Partition method, Error analysis of direct methods.

Iteration methods: Jacobi iteration method, Gauss-Siedal iterative method, SOR method. Convergence analysis of iterative methods.

Unit IV. Eigen Value Problem (11 L)

Eigen values and Eigen Vectors. Bounds on Eigen values, Jacobi's method for symmetric matrices, Given`s method for symmetric matrices, HouseHolder`s method for symmetric matrices, Rutihauser method for arbitrary matrices, Power method and inverse Power method.

Recommended Books

1. Kendall E. and Atkinson, An Introduction to Numerical Analysis, Wiley.
2. M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. S.D. Comte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGraw Hill International Book Company.
4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.
5. Hildebrand F.B., Introduction to Numerical Analysis, Dover Publication, NY.
6. Scarborough James B., Numerical Mathematical Analysis, Oxford University, Press, New Delhi.

Suggested Practicals :

1) Iteration methods based on first degree equation.
2) Iteration methods based on second degree equation.
3) Iteration methods for polynomial equations.
4) Linear System of equations.
5) Eigen values and Eigen Vectors-Jacobi method, Given`s method, HouseHolder`s method for symmetric matrices.
6) Eigen values and Eigen Vectors- Rutishauser method, Power method for arbitrary matrices.

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Mathematics Applied Component

5TH Semester

B.Sc. Mathematics — Course: S.MAT.5.AC

Title: COMPUTER PROGRAMMING - I

Learning Objectives:

1) To learn about OOP through java programming, applets.
2) Intro. to DBMS & RDBMS, SQL & PL/SQL Commands & Functions.

(3) Database Management Systems, Ramakrishnam, Gehrke, McGraw-Hill

(4) Ivan Bayross, "SQL,PL/SQL -The Programming language of Oracle", B.P.B. Publications, 3rd Revised Edition.

Unit I Introduction to Java Programming 15L

(a) Introduction: History of Java, Java features, different types of Java programs, Differentiate Java with C. Java Virtual Machine.
(b) Java Basics: Variables and data types, declaring variables, literals: numeric, Boolean, character and string literals, keywords, type conversion and casting. Standard default values. Java Operators, Loops and Controls (No Questions are to be asked on this topic).
(c) Classes: Defining a class, creating instance and class members: creating object of a class; accessing instance variables of a class; creating method; naming method of a class; accessing method of a class; overloading method; 'this' keyword, constructor and Finalizer: Basic Constructor; parameterized constructor; calling another constructor; finalize() method; overloading constructor.
(d) Arrays: one and two-dimensional array, declaring array variables, creating array objects, accessing array elements.
(e) Access control: public access, friendly access, protected access, private access.

Unit II Inheritance, Java Applets and Graphics Programming 15L

(a) Inheritance: Various types of inheritance, super and subclasses, keywords- 'extends'; 'super', overriding method, final and abstract class: final variables and methods; final classes, abstract methods and classes. Concept of interface. (b) Applets: Difference of applet and application, creating applets, applet life cycle, passing parameters to applets.
(c) Graphics, Fonts and Color: The graphics class, painting, repainting and updating an applet, sizing graphics. Font class, draw graphical figures - lines and rectangle, circle and ellipse, drawing arcs, drawing polygons. Working with Colors: Color methods, setting the paint mode.

Unit III Relational Database Management System 15L

(a) Introduction to Database Concepts: Database, Overview of database management system. Database Languages- Data Definition Language (DDL) and Data Manipulation Languages (DML).
(b) Entity Relation Model: Entity, attributes, keys, relations, Designing ER diagram, integrity constraints over relations, Conversion of ER to relations with and without constraints.
(c) SQL commands and Functions:
(i) Creating and altering tables: CREATE statement with constraints like KEY, CHECK, DEFAULT, ALTER and DROP statement.
(ii) Handling data using SQL: selecting data using SELECT statement, FROM clause, WHERE clause, HAVING clause, ORDER BY, GROUP BY,DISTINCT and ALL predicates, Adding data with INSERT statement, changing data with UPDATE statement, removing data with DELETE statement.

(iii) Functions: Aggregate functions-AVG, SUM, MIN, MAX and COUNT, Date functions- ADD_MONTHS(), CURRENT_DATE(), LAST_DAY(),

MONTHS_BETWEEN(), NEXT_DAY(). String functions LOWER(), UPPER(), LTRIM(), RTRIM(), TRIM(), INSTR(), RIGHT(), LEFT(), LENGTH(), SUBSTR(). Numeric functions: ABS(), EXP(), LOG(), SQRT(), POWER(), SIGN(), ROUND(number). (iv) Joining tables: Inner, outer and cross joins, union.

Unit IV Introduction to PL/SQL 15L

(a) Fundamentals of PL/SQL: Defining variables and constants, PL/SQL expressions and comparisons: Logical Operators, Boolean Expressions, CASE Expressions Handling, Null Values in Comparisons and Conditional Statements,
(b) PL/SQL Data types: Number Types, Character Types, Boolean Type. Datetime and Interval Types.
(c) Overview of PL/SQL Control Structures: Conditional Control: IF and CASE Statements, IF-THEN Statement, IF-THEN-ELSE Statement, IF-THEN-ELSIF Statement, CASE Statement,
(d) Iterative Control: LOOP and EXIT Statements, WHILE-LOOP, FOR-LOOP, Sequential Control: GOTO and NULL Statements.

Recommended Books:

(1) Programming with Java: A Primer 4th Edition by E. Balagurusamy, Tata McGraw Hill.
(2) Java The Complete Reference, 8th Edition, Herbert Schildt, Tata McGraw Hill.
(3) Database Management Systems, Ramakrishnam, Gehrke, McGraw-Hill.
(4) Ivan Bayross, "SQL,PL/SQL -The Programming language of Oracle", B.P.B. Publications, 3rd Revised Edition.

Additional References:

(a) Elsmasri and Navathe, "Fundamentals of Database Systems", Pearson Education.
(b) Peter Rob and Coronel, "Database Systems, Design, Implementation and Management", Thomson Learning
(c) C.J.Date, Longman, "Introduction to database Systems", Pearson Education.
(d) Jeffrey D. Ullman, Jennifer Widom, "A First Course in Database Systems", Pearson Education. (e) Martin Gruber, "Understanding SQL",B.P.B. Publications.
(f) Michael Abbey, Michael J. Corey, Ian Abramson, Oracle 8i – A Beginner's Guide, Tata McGraw-Hill.Eric Jendrock, Jennifer Ball, D Carson and others, The Java EE 5 Tutorial, Pearson
Education, Third Edition, 2003. (g)
Ivan Bayross, Web Enabled Commercial Applications Development Using Java 2, BPB Publications,Revised Edition, 2006
(h) Joe Wigglesworth and Paula McMillan, Java Programming: Advanced Topics, Thomson Course
Technology (SPD), Third Edition, 2004

(i) The Java Tutorials of Sun Microsystems Inc. http://docs.oracle.com/javase/tutorial.

Suggested Practicals:

1) Write a Java program to create a Java class: (a) without instance variables and methods, (b) with instance variables and without methods, (c) without instance variables and with methods, (d) with instance variables and methods
2) Write a Java program that illustrates the concepts of one, two dimension arrays.

3) Write a Java program that illustrates the concepts of Java class that includes (a) constructor with and without parameters (b) Overloading methods.
4) (a) Write a Java program to demonstrate inheritance by creating suitable classes.
(b)Write a Java applet to demonstrate graphics, Font and Color classes.
5) Creating a single table with/ without constraints and executing queries. Queries containing aggregate, string and date functions fired on a single table.
6) Updating tables, altering table structure and deleting table Creating and altering a single table and executing queries. Joining tables and processing queries.
7) Writing PL/SQL Blocks with basic programming constructs. 8) Writing PL/SQL Blocks with control structures.

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St. Xavier's College – Autonomous Mumbai

Syllabus For Even Semester Courses in MATHEMATICS (2018-2019)

Contents:

Theory Syllabus for Courses:

F.Y.B.Sc. – Mathematics **Course Code: S.MAT.2.01**

Title: CALCULUS – II

Learning Objectives: To learn about (i) Convergence of infinite series.
(ii) Intermediate Value Theorem and Mean Value Theorems. (iii) Applications of real valued differentiable functions of one variable.

Number of lectures : 45

Unit I: Series (15 Lectures)

Series of real numbers, simple examples of series, Sequence of partial sums, Convergence of series, convergent and divergent series, Necessary condition: series Σa_n is convergent implies $a_n \to 0$, converse not true, Algebra of convergent series, Cauchy criterion, $\Sigma\frac{1}{n^p}$ converges for p>1, divergence of $\Sigma\frac{1}{n}$, Comparison test, limit form of comparison test, Condensation test, Alternating series, Leibnitz theorem (alternating series test) and convergence of $\Sigma\frac{(-1)^n}{n}$, Absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test, Root test (without proofs) and examples. Tests for absolute convergence.

Unit II: Continuous functions and Differentiation (15 Lectures)

Properties of Continuous functions: If $f : [a,b] \to R$ is continuous at x_0 and $f(x_0) > 0$ then there exists a neighbourhood N of x_0 such that $f(x) > 0$ for all x in N. If $f : [a,b] \to R$ is continuous function then the image $f([a,b])$ isa closed interval, Intermediate value theorem and its applications, Bolzano-Weierstrass theorem (statement only), A continuous function on a closed and bounded interval is bounded and attains its bounds.

Differentiation of real valued function of one variable: Definition of differentiation at a point and on an open set, examples of differentiable and non-differentiable functions, differentiable functions are continuous but not conversely, Algebra of differentiable functions, chain rule, Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation
(only examples).

Unit III: Application of differentiation (15 Lectures)

Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave, convex functions, points of inflection, Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples, Monotone increasing and decreasing function, examples, L`hospital`s rule without proof, examples of indeterminate forms, Taylor's theorem with Lagrange's form of remainder with proof, Taylor polynomial and applications.

Recommended Books

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company,1994.
3. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd.

4. Robert G. Bartle and Donald R. Sherbet : Introduction to Real Analysis, Springer Verlag.
5. Howard Anton, Calculus - A new Horizon, Sixth Edition, John Wiley and Sons Inc,1999.

Additional Reference Books

1. Courant and John, A Introduction to Calculus and Analysis, Springer.
2. Ajit and Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
3. Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2000.
4. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
5. G. B. Thomas, Calculus, 12th Edition, 2009.

Assignments (Tutorials)

1. Calculating limit of series, Convergence tests.
2. Properties of continuous functions.
3. Differentiability, Higher order derivatives, Leibnitz theorem.
4. Mean value theorems and its applications.
5. Extreme values, increasing and decreasing functions.
6. Applications of Taylor's theorem and Taylors polynomials.

F.Y.B.Sc. – Mathematics **Course Code: S.MAT.2.02**

Title: Algebra II

Learning Objectives: To learn about (i) System of linear equations and matrices.
(ii) Vector Spaces
(iii) Basis and linear transformations.

Number of lectures : 45

Prerequisites:

Review of vectors in R^2 and R^3 as points, Addition and scalar multiplication of vectors in terms of co-ordinates, Dot product, Scalar triple product, Length (norm) of a vector.

Unit I: System of Linear equations and Matrices **(15 Lectures)**

Parametric equation of lines and planes, System of homogeneous and non-homogeneous linear equations, the solution of system of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for [m, n]=[1, 2], [1,3], [2,2], [2,3], [3,3]. Definition of n tuples of real numbers, sum of n tuples and scalar multiple of n tuple. Matrices with real entries, addition, scalar multiplication and multiplication of matrices, Transpose of a matrix, Type of matrices: zero matrix, identity matrix, scalar, diagonal, upper triangular, lower triangular, symmetric, skew-symmetric matrices, Invertible matrices, identities such as $[AB]^t = [B]^t [A]^t$, $[AB]^{-1}=[B]^{-1}[A]^{-1}$. System of linear equations in matrix form, elementary row operations, row echelon matrix, Gaussian elimination method, to deduce that the system of m homogeneous linear equations in n unknowns has a non-trivial solution if m < n.

Unit II: Vector spaces (15 Lectures)

Definition of real vector space, examples such as R^n with real entries, R[X]-space of m X n matrices over R, space of real valued functions on a non empty set. Subspace: Definition, examples of subspaces of R^2 and R^3 such as lines, plane passing through origin, set of 2 X 2 , 3 X 3 upper triangular, lower triangular, diagonal, symmetric and skew-symmetric matrices as subspaces of $M_2[R]$,$M_3[R]$, $P_n[X]$ of R[X] , solutions of m homogeneous linear equations in n unknowns as a subspace of R^n. Space of continuous real valued functions on a non-empty set X is a subspace of F[X,R]. Properties of subspaces such as necessary and sufficient condition for a non-empty subset to be a subspace of a vector space, arbitrary intersection of subspaces of a vector space is a subspace, union of two subspaces is a subspace if and only if one is a subset of the other, Linear combinations of vectors in a vector space, Linear span L[S] of a non-empty subset S of a vector space, S is the generating set of L[S], linear span of a non-empty subset of a vector space is a subspace of the vector space. Linearly independent / Linearly dependent sets in a vector space, properties such as a set of vectors in a vector space is linearly dependent if and only if one of the vectors v_i is a linear combination of the other vectors v_j's.

Unit III: Basis and Linear Transformation **(15 Lectures)** Basis of a vector space, Dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, minimal generating set of a vector space is a basis of a vector space, any set of n+1 vectors in a vector space with n elements in its basis is linearly dependent, any two basis of a vector space have the same number of elements, any n linearly independent vectors in an n dimensional vector space is a basis of a vector space. If U and W are subspaces of a vector space then U+W is a subspace of the vector space, dim [U+W]=dim U + dimW – dim [U ∩ W]. Extending the basis of a subspace to a basis of a vector space. Linear transformation, kernel, matrix associated with a linear transformation, properties such as kernel of a linear transformation is a subspace of the domain space, for a linear transformation T image [T] is a subspace of the co-domain space. If V, W are vector spaces with $\{v_1, \ldots, v_n\}$ basis of V and $\{w_1, \ldots, w_n\}$ are any vectors in W then there exists a unique linear transformation T such that $T(v_i) = w_i$. Rank- nullity theorem (only statement) and examples.

Recommended Books

1. Serge Lang, Introduction to Linear Algebra, Second Edition, Springer.
2. S. Kumaresan, Linear Algebra, A Geometric Approach, Prentice Hall of India, Pvt. Ltd, 2000.

Additional Reference Books

1. M. Artin: Algebra, Prentice Hall of India Private Limited, 1991.
2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi, 1971.
3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.
3. L. Smith: Linear Algebra, Springer Verlag.
4. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.
5. T. Banchoff and J. Warmers: Linear Algebra through Geometry, Springer Verlag, New York, 1984.
6. Sheldon Axler: Linear Algebra done right, Springer Verlag, New York.
7. Klaus Janich: Linear Algebra.
8. Otto Bretcher: Linear Algebra with Applications, Pearson Education.
9. Gareth Williams: Linear Algebra with Applications.

Assignments (Tutorials)

1. Solving homogeneous system of m equations in n unknowns by elimination for m , n = 1, 2, 1, 3 , 2, 2 , 2, 3 , 3, 3. Row echelon form.
2. Solving system AX= B by Gauss elimination, Solutions of system of linear equations.
3. Verifying whether V is a vector space for a given set V.
4. Linear span of a non-empty subset of a vector space, determining whether a given subset of a vector space is a subspace. Showing the set of convergent real sequences is a subspace of the space of real sequences etc.
5. Finding basis of a vector space such as $P_3[X]$, M_2 [R] etc. Verifying whether a set is a basis of a vector space. Extending basis to a basis of a finite dimensional vector space.
6. Verifying whether T : V → W is a linear transformation, finding kernel of a linear transformations and matrix associated with a linear transformation, verifying the Rank Nullity theorem.

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CIA I – 20 marks, 45 mins. (Objectives/Short questions, not more than 5 marks each)

CIA II – 20 marks, 45 mins. (Objectives/Short questions, not more than 5 marks each)

End Semester exam – 60 marks, 2 hours.

There will be three questions, one per unit. The Choice is internal- i.e. within a unit and could be between 50% to 100%

.Y.B.Sc. – Mathematics Course Code: S.MAT.4.01

Title: CALCULUS IV

Learning Objectives: (i) To learn about sequences in R^n and limit, continuity, differentiability, partial/ directional derivatives, gradients of scalar fields.
(ii) To study about limits, continuity, differentiability of scalar fields.
(iii) To learn about Second derivative test for extrema of functions of two variables and the method of Lagrange`s multipliers.

Number of lectures : 45

Unit I: Functions of several variables (15 Lectures)

1. Euclidean space, R^n– norm, inner product, distance between two points, open ball in R^n, definition of an open set / neighborhood, sequences in R^n, convergence of sequences–these concepts should be specifically discussed for n = 2 and n = 3.
2. Functions from $R^n \to R$ (Scalar fields) and from $R^n \to R^n$ (Vector fields). Iterated limits, limits and continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of vector fields.
3. Directional derivatives and partial derivatives of scalar fields.
4. Mean value theorem for derivatives of scalar fields.

Reference for Unit I:
(1) T. Apostol, Calculus, Vol. 2, John Wiley.
(2) J. Stewart, Calculus, Brooke/Cole Publishing Co.

Unit II: Differentiation (15 Lectures)

1. Differentiability of a scalar field at a point (in terms of linear transformation) and in an open set, Total derivative, Uniqueness of total derivative of a differentiable function at a point. (Simple examples of finding total derivative of functions such as $f(x, y) = x^2 + y^2$, $f(x, y, z) = x + y + z$ may be taken). Differentiability at a point implies continuity, and existence of directional derivative at the point. The existence of continuous partial derivatives in neighborhood of point implies differentiability at the point.
2. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.
3. Chain rule for scalar fields.
4. Higher order partial derivatives, mixed partial derivatives. Sufficient condition for equality of mixed partial derivative.

Reference for Unit II:
(1) Calculus, Vol. 2, T. Apostol, John Wiley.
(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.

Unit III: Applications (15 Lectures)

1. Second order Taylor's formula for scalar fields.
2. Differentiability of vector fields, definition of differentiability of a vector field at a point, Hessian /Jacobian matrix, differentiability of a vector field at a point implies continuity, the chain rule for derivative of vector fields (statement only).
3. Mean value inequality.
4. Maxima, minima and saddle points.
5. Second derivative test for extrema of functions of two variables.
6. Method of Lagrange`s multipliers.

Reference for Unit III:
sections 9.9, 9.10, 9.11, 9.12, 9.13, 9.14 from T. Apostol, Calculus Vol. 2, John Wiley.

Suggested Tutorials:

1. Sequences in R^2 and R^3, limits and continuity of scalar fields and vector fields using "definition" and otherwise iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Taylor's formula, differentiation of a vector field at a point, finding Hessian/ Jacobian matrix, Mean value inequality.
6. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two/three variables and method of Lagrange`s multipliers.

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S.Y.B.Sc. – Mathematics   Course Code: S.MAT.4.02

Title: ALGEBRA–IV

Learning Objectives: (i) To learn properties of groups and subgroups
(ii) To study cyclic groups and cyclic subgroups
(iii)To understand Lagrange's theorem and Group homomorphisms and isomorphisms.

Number of lectures : 45

Unit I: Groups and subgroups (15 Lectures)

(a) Definition of a group, abelian group, order of a group, finite and infinite groups.

Examples of groups including

(i) Z, Q, R, C under addition

(ii) Q*(=Q\{0}), R*(=R\{0}), C*(=C\{0}), Q^+ (positive rational numbers) under multiplication

(iii) Z_n – the set of residue classes modulo n under addition

(iv) U(n) – the group of prime residue classes modulo n under multiplication (v) The symmetric group S_n

(vi) The group of symmetries of plane figure.The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for n = 3, 4)

(vii) Klein 4 – group

(viii)Matrix groups $M_{\times}$ (R) under addition of matrices; GL_n(R) – the set of invertible real matrices under multiplication of matrices.

(ix) Examples such as S^1 as a subgroup of C , μ – the subgroup of n^{th} roots of unity.

Properties such as

1) In a group (G, .), the following indices rules are true for all integers n, m:-

(i) $a^n a^m = a^{n+m}$ for all a in G

(ii) $(a^n)^m = a^{nm}$ for all a in G

(iii) $(ab)^n = a^n b^n$ for all a, b in G whenever ab = ba 2) In a group (G, .), the following are true:- (i) The identity element e of G is unique.

(ii) The inverse of every element in G is unique.

(iii) $(a^{-1})^{-1} = a$

(iv) $(ab)^{-1} = b^{-1} a^{-1}$

(v) if $a^2 = e$ for every a in G then (G, .) is an abelian group

(vi) if $(aba^{-1})^n = ab^n a^{-1}$ for every a, b in G and for every integer n

(vii) if $(ab)^2 = a^2 b^2$ for every a, b in G then (G, .) is an abelian group

(viii) $(Z^*, .)$ is a group if and if n is prime

3) Properties of order of an element such as (n and m are integers) (i) Let o(a) = n. Then $a^m = e$ if and only if n| m

(ii) If o(a) = nm then $o(a^n) = m$.

(iii) If o(a) = n then $o(a^m) = \frac{\quad}{(\; , \;)}$ where (n, m) is GCD of n and m

(iv) $o(aba^{-1}) = o(b)$, o(ab) = o(ba)

(v) If o(a) = n, o(b) = m, ab = ba, (n, m) = 1 then o(ab) = nm. (b) Subgroups

1) Definition, necessary and sufficient condition for a non-empty set to be a Subgroup
2) The center Z(G) of a group G is a subgroup.
3) Intersection of two (or a family of) subgroups is a subgroup.
4) Union of two subgroups is not a subgroup in general. Union of two sub groups is a subgroup if and only if one is contained in the other.
5) Let H and K are subgroups of a group G. Then HK is a subgroup of G if and only if HK = KH.

Reference for Unit I:
(1) I.N. Herstein, Topics in Algebra.
(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit II: Cyclic groups and cyclic subgroups (15 Lectures)
(a) Cyclic subgroup of a group, cyclic groups, (examples including Z, Zn, μ).
(b) Properties such as
i) Every cyclic group is abelian ii) Finite cyclic groups, infinite cyclic groups and their generators iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group. iv) Subgroup of a cyclic group is cyclic.
v) In a finite group G, G = <a> if and only if o(G) = o(a).
vi) Let G = <a> and o(a) = n. Then G = $< a^m >$ if and only if (m, n) = 1.
vii) If G is a cyclic group of order p^n and H<G, K<G then prove that either H⊆K or K⊆H

Reference for Unit II:
(1) I.N. Herstein, Topics in Algebra.
(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit III: Lagrange's Theorem and Group homomorphism (15 Lectures) a) Definition of a Coset and properties such as
1) If H is a subgroup of group G and x∈G then prove that
(i) xH = Hx if and only if x∈G (ii) Hx = H if and only if x∈G
2) If H is a subgroup of group G and x, y∈G then prove that
(i) xH = yH if and only if $x^{-1}y$ ∈H (ii) Hx = Hy if and only if xy^{-1} ∈H
3) Lagrange's theorem and consequences such as Fermat's Little theorem, Eulers's theorem.

If a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic.

b) Group homomorphisms and isomorphisms, automorphisms
1) Definition
2) Kernel and image of a group homomorphism.
3) Examples including inner automorphism.

Properties such as
(i) If f : G → G` is a group homomorphism then Ker f < G.
(ii) Let f : G → G` be a group homomorphism. Then Ker f ={e} if and only if f is 1-1.

(iii) Let $f : G \to G`$ be a group isomorphism. Then
G is abelian/cyclic if and only if G` is abelian/cyclic

Reference for Unit III:
(1) I.N. Herstein, Topics in Algebra.
(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books:
1. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
2. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
5. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books:
1. S. Adhikari, An Introduction to Commutative Algebra and Number theory, Narosa Publishing House.
2. T.W. Hungerford. Algebra, Springer.
3. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luther, I.B.S. Passi. Algebra, Vol. I and II.

Suggested Tutorials:
1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle, square.
3. Subgroups.
4. Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.
6. Group homomorphisms, isomorphisms.

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S.Y.B.Sc. – Mathematics Course Code: S.MAT.4.03

Title: DIFFERENTIAL EQUATIONS

Learning Objectives: (i) To learn First order First degree Ordinary Differential equations
(ii) To study Second order Ordinary Linear Differential equations
(iii)To learn how to solve Partial Differential equations Number of lectures : 45

Unit I: First order First degree Ordinary Differential equations (15 Lectures)

1. Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non linear ODE.

2. Existence and Uniqueness Theorem for the solutions of a second order initial value problem (statement only). Define Lipschitz function; solve examples verifying the conditions of existence and uniqueness theorem.
3. Review of solution of homogeneous and non- homogeneous differential equations of first order and first degree. Notion of partial derivative. Exact Equations: General Solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations. Rules for finding integrating factors (without proof) for non exact equations, such as

   $\frac{1}{Mx+Ny}$ (i) is an I.F. if $Mx + Ny \neq 0$ and $Mdx + Ndy$ is homogeneous.

   (ii) is an I.F. if $Mx - Ny \neq 0$ and $Mdx - Ndy$ is of the type $f(x, y)y\, dx + f(x, y)x\, dy$.

   (iii) $e^{\int}$ ( ) is an I.F. if $N \neq 0$ and $-(-\ -$ $-$ is a function of x alone, say $f(x)$.

   (iv) $e^{\int}$ ( ) is an I.F. if $M \neq 0$ and $-(-\ -)$ $-$ is a function of y alone, say $g(y)$.
4. Linear and reducible to linear equations, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

Unit II: Second order Ordinary Linear Differential equations (15 Lectures)

1. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equation. The use of known solutions to find the general solution of homogeneous equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.
2. The homogeneous equation which constant coefficient. auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.
3. Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Unit III: Partial Differential equations (15 Lectures)

1. Classification of Second Order Partial Differential Equations
2. One-Dimensional Wave Equation
   (i) Vibration of an Infinite String
   (ii) Vibration of a Semi-infinite String
   (iii)Vibration of a Finite String
3. Laplace Equation
   (i) Green's Equation

4. Heat Conduction problem (i)
   Infinite Rod Case
   (ii) Finite Rod Case

Recommended Books for Unit – III:

1. An Elementary Course in Partial Differential Equations – T. Amarnath, Narosa Publishing House
2. Differential Equations with Applications and Historical Notes – G. F. Simmons – McGraw Hill.

References:

1. Differential equations with applications and historical notes- G. F. Simmons-McGraw Hill.
2. An introduction to ordinary differential equations - E. A. Coddington.
3. Differential Equations-Shepley L. Ross-Wiley.
4. Mathematical Modeling with Case Studies, A Differential Equation Approach Using Maple- Belinda Barnes and Glenn R. Fulford-Taylor and Francis.
5. Differential Equations and Boundary Value Problems: Computing and Modeling-C. H. Edwards and D. E. Penny-Pearson Education.
6. Linear Partial Differential Equation for Scientists and Engineers-Tyn Myint-U and Lokenath Debnath-Springer.
7. Partial Differential Equations: An Introduction with Mathematica and MAPLE- Ioannis P Stavroulakis and Stepan A Tersian.
8. Ordinary and Partial Differential Equations-M.D.Raisinghania-S.Chand.

Suggested Practicals:

1. Application of existence and uniqueness theorem, solving exact and non exact equations.
2. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.
3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4. Solving equations using method of undetermined coefficients and method of variation of parameters.
5. Determining whether a given second order linear partial differential equation is elliptic, parabolic or hyperbolic.
6. Using method of separation of variables for different equations including heat equation and Laplace equation.

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T.Y.B.Sc. - Mathematics Course Code: S.MAT.6.01

Title of Paper: CALCULUS - VI

Learning Objectives:
After completion of the course, a student should:
(i) Learn about sequences and series of real and complex functions.
(ii) Find power series representations of functions.
(iii) Find Laurent series of a function in the neighbourhood of isolated singularity. (iv) Learn about analytic functions and understand the difference between differentiable and analytic functions.

Number of lectures: 45

Unit I: Sequences and Series of Functions (12 Lectures)

Sequence of functions-pointwise and uniform convergence of sequences of real-valued functions, examples.
Uniform convergence implies pointwise convergence, example to show converse not true. Series of functions, convergence of series of functions, Weierstrass M-test. Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval. Examples.
Consequences of these properties for series of functions, term by term differentiation and integration.

Unit II: Power Series (11 Lectures)

Limit superior and Limit inferior.
Power series in $\mathbb{R}$ centered at origin and at some point in $\mathbb{R}$, radius of convergence, region (interval) of convergence, uniform convergence, term-by-term differentiation and integration of power series, Examples.
Cauchy Hadamard Theorem. Abel's Theorem.
Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Unit III: Introduction to Complex Analysis (11 Lectures)

Review of Complex Numbers: Complex Plane, Polar Coordinates, Exponential Map, Powers and roots of Complex numbers. De Moivre's formula, $\mathbb{C}$ as a metric space, Bounded and Unbounded Sets, point at infinity-extended complex plane, Sketching of set in complex plane.
Limit at a point, theorems on limit, convergence of sequences of complex numbers and results using properties of real sequences.
Functions: $\mathbb{C} \to \mathbb{C}$, real and imaginary part of functions, continuity at a point and algebra of continuous functions.

Derivatives of: $\mathbb{C} \to \mathbb{C}$, comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient condition for differentiability, analytic functions, conjugate of an analytic function is analytic, chain rule.
Harmonic functions and harmonic conjugates.
Mobius Transformations – Definition and examples.

Unit IV: Complex Power Series (11 Lectures)

Definite integrals of functions.
Cauchy Integral Formula, Derivatives of Analytic function.
Taylor's Theorem for analytic functions.
Exponential Functions and its properties, Trigonometric Functions, Hyperbolic Functions. Power series of complex numbers and related results following from Unit II, radius of convergences, disc of convergence, uniqueness of series representation, examples. Definition of Laurent series, Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, statement of residue theorem and calculation of residue.

References:

1. Richard R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi.
2. Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis.
3. J. W. Brown and R. V. Churchill, Complex Variables and Applications.

Additional References:

1. Robert G. Bartle and Donald R. Sherbert, Introduction to Real Analysis.
2. Charles G. Denlinger, Elements of Real Analysis, Jones and Bartlett (Student Edition),2011.
3. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable.
4. Theodore W. Gamelin, Complex analysis, Springer.
5. Joseph Bak and Donald J. Newman, Complex analysis (2nd Edition), Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., New York, 1997.
6. Murray R. Spiegel, John Schiller and Seymour Lipschutz, Complex Variables, Schaum`s Outline series McGraw-Hill Book Company, Singapore.
7. John B. Conway, Functions of one complex variable, Springer, Second edition.
8. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, New Delhi.
9. Lars V. Ahlfors, Complex Analysis, Mc-Graw Hill Education.
10. Dennis G. Zill, Patrick Shanahan, Complex Analysis - A first course with Applications.

Suggested Practicals (3 practicals per batch per week):

1. Pointwise and uniform convergence of sequence functions, properties.

2. Point wise and uniform convergence of series of functions and properties.
3. Limit continuity and derivatives of functions of complex variables.
4. Analytic function, finding harmonic conjugate, Mobius transformations.
5. Cauchy integral formula, Taylor series, Power Series
6. Finding isolated singularities- removable, pole and essential, Laurent series, Calculation of residue.

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T.Y.B.Sc. – Mathematics Course Code: S.MAT.6.02

Title: ALGEBRA VI

Learning Objectives (i)To learn Normal Subgroups and Classification of groups.

(ii) To learn Ring theory.

Unit I. Group Theory (12L)

Review of Groups, Subgroups, Abelian groups, Order of a group, Finite and infinite groups, Cyclic groups, The Center Z(G) of a group G, Group homomorphisms, isomorphisms, automorphisms, inner automorphisms (No question be asked),Normal subgroups: Normal subgroups of a group, definition and examples including center of a group, Quotient group, Alternating group A_n, Cycles. Listing normal subgroups of A_4, S_3. First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups), Second Isomorphism theorem, third Isomorphism theorem, Cayley's theorem, External direct product of direct products, Order of an element in a direct product, criterion for direct product to be cyclic, Classification of groups of order ≤ 7.

Reference for Unit I:

(i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

Unit II. Ring Theory (11L)

Motivation: Integers & Polynomials.Definitions of a ring (The definition should include the existence of a unity element),zero divisor, unit, the multiplicative group of units of a ring. Basic Properties & examples of rings, including $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, Mn($\mathbb{R}$), $\mathbb{Q}$[X], $\mathbb{R}$[X], $\mathbb{C}$[X], $\mathbb{Z}$[i], $\mathbb{Z}[\sqrt{2}]$, $\mathbb{Z}[\sqrt{-5}]$,$\mathbb{Z}$n. Definitions of Commutative ring, integral domain (ID), Division ring, examples. Theorem such as: A commutative ring R is an integral domain if and only if for a, b, c ∈ R with a ≠ 0 the relation ab = ac implies that b = c. Definitions of Subring, examples. Ring homomorphisms, Properties of ring homomorphisms, Kernel of ring homomorphism, Ideals, Operations on ideals and Quotient rings, examples. Factor theorem and First and second Isomorphism theorems for rings, Correspondence Theorem for rings. Definitions of characteristic of a ring, Characteristic of an ID. Definition of field, subfield and examples, characteristic of fields. Any field is an ID and a finite ID is a field. Definitions of characteristic of a ring, Characteristic of an ID. Definition of field, subfield and examples, characteristic of fields.

Reference for Unit II:

(i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

Unit III. Polynomial Rings and Field theory (11L)

Principal ideal, maximal ideal, prime ideal, the characterization of the prime and maximal ideals in terms of quotient rings. Polynomial rings, R[X] when R is an integral domain/ Field. Divisibility in Integral Domain, Definitions of associates, irreducible and primes. Prime (irreducible)elements in $\mathbb{R}$[X], $\mathbb{Q}$[X], $\mathbb{Z}_p$[X]. Eisenstein's criterion for irreducibility in $\mathbb{Z}$.

Reference for Unit III:

(i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.
(iii) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited

Unit IV: E.D. , P.I.D., U.F.D. (11 L)

Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique Factorization Domain (UFD). Examples of ED: Z, F[X], where F is a field, and Z[i]. (ii) An ED is a PID, a PID is a UFD. (iii) Prime (irreducible) elements in R[X], Q[X], Zp[X]. Prime and maximal ideals in R[X], Q[X]. (iv) Z[X] is a not a UFD. Prime and maximal ideals in polynomial rings. Characterization of fields in terms of maximal ideals, irreducible polynomials. Construction of quotient field of an integral domain (Emphasis on Z, Q). A field contains a subfield isomorphic to Z_p or Q.

Reference for Unit IV:

(i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi. (iii) P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.

Recommended Books:

i) N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
(ii) J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.
(iii) N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited
(iv) P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995

Additional Reference Books

(i) M. Artin, Algebra, Prentice Hall of India, New Delhi.
(ii) J. B. Fraleigh, A First course in Abstract Algebra, Third edition, Narosa, New Delhi
(iii) D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
(iv)T.W. Hungerford. Algebra, Springer.
(v) I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.

Suggested Practicals:

1) Rings, Subrings, Ideals, Ring Homomorphism and Isomorphism

2) Prime Ideals and Maximal Ideals

3) Polynomial Rings , Fields

4) E.D, P.I.D,U.F.D.

5) Normal Subgroups and quotient groups.

6) Cayley's Theorem and external direct product of groups.

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Topology of metric spaces II

Course Code : S.MAT.6.03

Course Objective : Introduction to completeness and connectedness in metric spaces.

Unit I. Complete Metric Spaces (10 L)

Definition of complete metric spaces, Examples of complete metric spaces.
Completeness property in subspaces. Nested Interval theorem in $\mathbb{R}$. Cantor's intersection theorem.

Unit II. Applications Of Cantor's Intersection theorem (15 L)

Cantor's intersection theorem. Applications of Cantor's intersection theorem:

The set of real numbers is uncountable. Density of rational numbers in real numbers.

Bolzano- Weierstrass Theorem: Every bounded sequence of real numbers has a convergent subsequence.
Intermediate Value Theorem : Let f :[a, b] $\rightarrow \mathbb{R}$ be continuous and assume that f (a) and f(b) have opposite signs, say f(a)< 0 and f(b)>0. Then there exists c (a, b) such that f(c) =0.

Let I=[a,b] be a closed and bounded interval. Let $\{J_{\alpha} : \alpha \in \Lambda\}$ be a family of open intervals such that I $\subset \bigcup_{\alpha\in\Lambda} J_\alpha$. Then there exists a finite subset F $\subset \Lambda$ such that I$\subset \bigcup_{\alpha\in F} J_\alpha$, that is, I is contained in the union of a finite number of open intervals of the given family. Finite intersection property of closed sets for compact metric space, hence every compact metric space is complete.

Unit III. Continuous functions on metric spaces (10 L)

Epsilon-delta definition of continuity at a point of a function from one metric space to another, Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples. Algebra of continuous real valued functions on a metric space, Continuity of composite of continuous functions, Continuous image of compact set is compact. Uniform continuity in a metric space, definition and examples (emphasis on $\mathbb{R}$). Contraction mapping and fixed point theorem, Applications.

Unit IV. Connected sets (10 L)

Separated sets- definition and examples, disconnected sets, disconnected and connected metric spaces, connected subsets of a metric space. Connected subsets of $\mathbb{R}$, A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected, Characterization of a connected space, viz. A metric space is connected if and only if every continuous function from X to {-1, 1} is a constant function. Path connectedness, definition and examples, A path connected subset is connected, convex sets are path connected, Connected components, An example of a connected set which is not path connected.

Reference Books:

1. S. Kumaresan, Topology of Metric spaces.
2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.

Additional Reference Books.

1. W. Rudin, Principles of Mathematical Analysis.
2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974
3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.
4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.
5. P. K. Jain. K. Ahmed. Metric Spaces. Narosa, New Delhi, 1996.
6. W. Rudin. Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.
7. D. Somasundaram, B. Choudhary. A first Course in Mathematical Analysis. Narosa, New Delhi
8. G.F. Simmous, Introduction to Topology and Modern Analysis, McGraw-Hii, New York, 1963.
9. Sutherland. Topology

Reference for Units I and II: Expository articles of MTTS programme

Suggested Practicals:

1) Examples of complete metric spaces

2) Cantor's theorem and applications
3) Continuous functions on metric spaces
4) Uniform continuity and fixed point theorem
5) Examples of connected sets and connected metric spaces
6) Path connectedness, convex sets , equivalent condition for connected set using continuous function

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T.Y. B.Sc. Mathematics Course Code: S.Mat.6.04

Title: Numerical Methods-II

Learning Objectives: To learn about (i) different interpolation methods.
(ii) polynomial approximations.
(iii) numerical differentiation and Numerical Integration
(iv) solving Ordinary Differential Equations(IVP)

Number of lectures: 45

Unit I. Interpolation (12 L)

Interpolating polynomials, Uniqueness of interpolating polynomials. Linear, Quadratic and higher order interpolation. Lagrange's Interpolation. Newton`s divided interpolation. Finite difference operators: Shift operator, forward, backward and central difference operator, Average operator and relation between them. Difference table, Relation between difference and derivatives. Fundamental theorem of difference calculus. Factorial notation. Interpolating polynomials using finite differences Gregory-Newton forward difference interpolation, Gregory-Newton backward difference interpolation, Stirling's Interpolation. Results on interpolation error.

Unit II. Polynomial Approximations and Numerical Differentiation (12 L)

Hermite Interpolation, Piecewise Interpolation: Linear, Quadratic and Cubic.Bivariate Interpolation: Lagrange's Bivariate Interpolation, Newton's Bivariate Interpolation. Least square approximation.

Numerical differentiation: Numerical differentiation based on Interpolation, Numerical differentiation based on finite differences (forward, backward and central), Numerical Partial differentiation.

Unit III. Numerical Integration (11 L)

Numerical Integration based on Interpolation. Newton-Cotes Methods, Trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule. Determination of error term for all above methods. Convergence of numerical integration: Necessary and sufficient condition (with proof). Composite integration methods; Trapezoidal rule, Simpson's rule.

Unit IV. Ordinary Differential Equations (IVP) (10 L) Single step methods: Taylor`s series method, Picard`s method of successive approximations, Euler`s

Method (with geometrical interpretation and error analysis), Modified Euler`s method(with geometrical interpretation and error analysis), Runge-Kutta method of second and fourth order (with geometrical interpretation and error analysis)

Recommended Books

1. Kendall E. and Atkinson, An Introduction to Numerical Analysis, Wiley.
2. M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. S.D. Comte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGraw Hill International Book Company.
4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.
5. Hildebrand F.B., Introduction to Numerical Analysis, Dover Publication, NY.
6. Scarborough James B., Numerical Mathematical Analysis, Oxford University, Press, New Delhi.

Suggested Practicals :

1) Lagrange's Interpolation, Newton`s divided interpolation,
2) Gregory-Newton forward/backward difference Interpolation and Stirling Interpolation.
3) Hermite Interpoaltion, Piece-wise interpolation, Bivariate Interpolation, Least Square approximation
4) Numerical differentiation: Finite differences (forward, backward and central), Numerical Partial differentiation
5) Numerical Integration: Trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule. Composite integration methods: Trapezoidal rule, Simpson's rule.
6) Ordinary differential equations(IVP) by Taylor series method, Picard`s method, Euler`s method, Modified Euler`s method, Runge Kutta method.

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T.Y.B.Sc. Syllabus Under Autonomy

Mathematics Applied Component

6^{TH} Semester

B.Sc. Maths Course: S.MAT.6.AC

Learning Objectives – The objective of this course is to introduce various concepts of programming to the students using Python.

Expected Learning Outcomes of this course:

1) Students should be able to understand the concepts of programming before actually starting to write new programs. .
2) Students should be able to understand what happens in the background when the programs are executed
3) Students should be able to develop logic for Problem Solving.

4) Students should be made familiar about the basic constructs of programming such as data, operations, conditions, loops, functions etc.
5) Students should be able to apply the problem solving skills using syntactically simple language i.e. Python (version: 3.X or higher)

Unit I 15L

Introduction to Programming Languages : What is program and programming paradigms, Programming languages-their classification and characteristics, language translators and language translation activities, Use of Algorithms/Flow Charts for problem solving.
Building Blocks of Program: Data, Data Types, Data Binding, Variables, Constants, Declaration, Operations on Data such as assignment, arithmetic, relational, logical operations, dry run, Evaluating efficiency of algorithms in terms of number of operations and variables used.

Unit II 15L

Introduction to Python Programming: Features, basic syntax, Writing and executing simple program, Basic Data Types such as numbers, strings, etc Declaring variables, Performing assignments, arithmetic operations, Simple input-output.
Sequence Control – Precedence of operators, Type conversion.

Conditional Statements: if, if-else, nested if –else.

Looping: for, while, nested loops.
Control statements: Terminating loops, skipping specific conditions.

Unit III 15L

String Manipulation: declaring strings, string functions.
Manipulating Collections Lists, Tuples.
Dictionaries – Concept of dictionary, techniques to create, update &delete dictionary items.

Unit IV 15L

Functions: Defining a function, calling a function, Advantages of functions, types of functions, function parameters, Formal parameters, Actual parameters, anonymous functions, global and local variables.
Modules: Importing module, Creating & exploring modules, Math module, Random module, Time module.
Recommended Books:

1) Charles Dierbach, Introduction to Computer Science using Python, Wiley, 2013
2) James Payne , Beginning Python: Using Python 2.6 and Python 3, Wiley India, 2010

Additional References:

1. Paul Gries, Jennifer Campbell, Jason Montojo, Practical Programming: An Introduction to Computer Science Using Python 3, Pragmatic Bookshelf, 2/E 2014
2. Adesh Pandey, Programming Languages – Principles and Paradigms, Narosa, 2008
3. Mark Lutz, Learning Python, O'Reilly Media; 5 edition (2013)
4. https://docs.python.org/3/tutorial
5. https://www.guru99.com/python-tutorials.html
6. https://www.python-course.eu

Suggested practicals:

(1) Simple programs like printing the names, numbers, mathematical calculations, etc.
(2) Simple programs containing variable declaration and arithmetic operations.
(3) Programs based on conditional constructs.
(4) Programs based on loops.
(5) Programs related to string manipulation.
(6) Programs related to Lists, Tuples.
(7) Programs related to dictionaries.
(8) Programs related to functions & modules.

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